\[ \frac{d(g_K + \delta)}{d\pi} = \beta \frac{dv}{d\pi} \]

If \( \eta_w = 0 \), we see that:
\[ \frac{dv}{d\pi} = -\eta_w \frac{\rho}{\beta \pi \rho - \eta_w} < 0 \]

If \( \eta_w = 0 \), on the other hand:
\[ \frac{dv}{d\pi} = \frac{\eta_p}{\beta} > 0 \]

Thus the impact of an increase in profit share on growth depends on the coefficients of the investment function, leading to the distinction between wage-led and profit-led growth regimes.

Chapter 11

Land-Limited Growth

11.1 Non-Reproducible Resources

In the Classical conventional wage model where labor-power is supplied elastically at a given wage, all inputs are reproducible. The economy can produce capital by itself, and, practically speaking, it can reproduce labor-power as well by paying the wage. In this type of model, there are no resource limitations to growth. The growth rate of the economy is determined entirely by productivity and the propensity of capitalists to accumulate.

When labor-power is inelastically supplied and grows at an exogenously given rate, as in the Solow–Swan model, the forces determining the steady-state growth rate change sharply. The long-run growth rate of the economy has to adjust to the given natural growth rate of the labor force. Input prices must vary to make this adjustment.

In this chapter we will study a Ricardian economy where there is a fixed and limited amount of land which is necessary for production. We will suppose that property rights in land exist, creating both a rental market for the productive use of land in each period, and a land market on which land can be bought and sold.

Capitalists' asset portfolios now include both capital and land, and their portfolio choices determine a price for land. Since capitalists can invest either in capital or in land, the returns from owning land and their expectations about the path of the future price of land play a central role in the economy. The introduction of this second asset raises issues of asset pricing that are fundamental to the modern theory of finance as well.
11.2 Ricardo’s Stationary State

David Ricardo, a successful London stockbroker whose impecably logical analysis of economic growth and distribution was a major influence in the development of political economy, analyzed the growth of an economy with limited land in his *Principles of Political Economy and Taxation*.

Ricardo works with a *corn model* of production very similar to the Classical model that we presented earlier. In this model he abstracts from the real diversity of commodities and assumes that the only produced good is corn (the comprehensive English term for all food grains), which he will take as the monétaire. Production of corn requires workers, whose wages must be paid during the production period between the planting of the corn and its harvesting. These advanced wages, together with the seed corn, constitute the capital required to carry on production. Ricardo follows Malthus in assuming that the wage (in terms of corn) is fixed at a level where the birth and death rates of the population are close to equal, as in Chapter 4. In our terms Ricardo’s corn model is a conventional wage model.

In Ricardo’s world there are three classes: workers, capitalists, and landowners. The capitalists rent land from the landowners and perform the entrepreneurial function of organizing production, as well as the capitalist function of owning and accumulating capital. The landowners own the land and rent it to the capitalists.

The central economic idea of Ricardo’s model is that different plots of land have different natural fertilities. Conceptually we divide up all the land in the economy into plots that require the same dose of capital and labor to cultivate (though they may not all have the same actual area). Each plot of land has a certain *yield*, the average harvest that can be expected when the standard dose of labor and capital of standard quality are applied to it. Ricardo imagines that we can rank all the land in the economy according to its fertility, starting from the most fertile, and proceeding to the least fertile. If we graph the plots of land along the horizontal axis and the yield of each plot of land on the vertical axis, as in Figure 11.1, we can visualize the *diminishing returns* that are at the heart of Ricardo’s thinking as a *marginal product schedule* of capital and labor applied together in fixed proportions. Since each plot of land requires the same amount of capital and labor, the distance along the horizontal axis measures the labor force (which Ricardo takes as proportional to population) and capital employed. The yield of the least fertile land in cultivation (the *extensive margin* in Ricardo’s language) is the marginal product of capital and labor together; since the removal of one dose of capital and labor would remove the marginal land from cultivation and reduce the total output by its yield. The area under the marginal product schedule up to the extensive margin is the total corn output of the economy.

On this graph we can draw a horizontal line at the height of the real wage, determined as to bring about a demographic equilibrium of births and deaths. This is the familiar conventional wage supply schedule of labor. The amount of capital accumulated in the past by capitalists can offer employment for a certain number of workers, and thus determines the population, as well as the amount of land in cultivation and the extensive margin, in Ricardo’s framework. The area under the marginal product schedule above the real wage is the surplus product of the economy, since it represents the excess of production over what is necessary to keep the current population of workers alive.

This surplus is divided between landowners and capitalists as they bargain over the rents on the various plots of land. Ricardo argued that the owner of the marginal land in cultivation could charge only a nominal rent on her land, since other land almost as good is available and earning no rent at all. He also argued that competition among the capitalists would force the rent on any cultivated plot of land to the point where the capitalist’s profit rate on that land after paying rent would just be equal to the profit rate on the marginal land that paid practically no rent. Thus in Figure 11.1 the horizontal line at the level of the yield of the marginal land divides the surplus into rent and profit. Profit is the rectangle between the wage line and the yield on the extensive margin, and rent is the area above the yield on the marginal land under the marginal product schedule.

Ricardo assumed, extrapolating the behavior he thought he observed in British society, that workers and landowners would spend all their income on consumption (workers on wages for subsistence, and landowners on staffing great houses with armies of liveried servants), and that capitalists would save all or most of their profit income, and accumulate it to expand production. As long as profits are positive, capital will be expanding, creating more jobs, supporting a larger population, and pushing the extensive margin into less fertile land. As this accumulation occurs, the wage remains constant, but the profit rate falls.

In the end, Ricardo predicts that capital accumulation and growth will stop when the extensive margin is pushed out to the point where the yield on the marginal land is just equal to the conventional wage. At this point the profit rate and total profits are zero, so there is no more capital accumulation, and all the surplus takes the form of rent. Ricardo called this situation the *stationary state*. At the stationary state most of a large population lives at the edge of subsistence, pressing on the limited resources of the earth, while a small wealthy class of landowners
appropriate the social surplus product.

Ricardo's analysis of the stationary state has strong echoes in contemporary anxieties about resource depletion and environmental degradation as a result of economic growth. The diminishing returns that Ricardo modeled in terms of land could be seen as arising from the exhaustion of non-renewable resources and the destruction of the environment, and the stationary state as the unhappy fate awaiting an overcrowded humanity on a finite planet.

11.3 Production with Land

Let us consider the one-sector model of production, but with the added requirement that land is required as an input. In this chapter we will assume that the technology of production is unchanging over time. We will use the letters $U$ and $u$ for land and land per worker (since $L$ is easy to confuse with labor). The technique of production then becomes:

$$\text{1 labor} + k \text{ capital} + u \text{ land} \rightarrow x \text{ output} + (1 - \delta)k \text{ capital} + u \text{ land}$$

In other words, in this model one worker equipped with $k$ units of capital and $u$ units of land can produce $x$ units of output at the end of a year. Capital depreciates at the rate $\delta$, but land does not depreciate at all. We are free to measure land in any units (acres or hectares, for example), and the coefficient $u$ will change proportionately. In order to simplify the equations of the model, we will take as our unit of land the amount of land required per unit of capital, so that $u = k$. Thus we can measure land and capital directly on the same scale.

In any period there is a fixed amount of capital, $K_t$, inherited from the past, and a fixed amount of land $U$. Unlike Ricardo's model, all the land here is assumed to be of the same fertility. The quantity of capital will change from period to period because the economy can produce capital and capitalists can accumulate it. The quantity of land, on the other hand, can never change.

The idea that there is a fixed resource limitation of some kind (like land in this model) is very strongly rooted in human attitudes towards economic growth, but is perhaps not very well confirmed by human experience. First of all, all resources require some development. Agricultural land must be cleared, drained and plowed. Mineral resources must be discovered and developed (through the construction of mines, wells, and transportation facilities). Second, the process of technical change frequently renders resources obsolete before they are exhausted. The iron deposits in the Eastern United States that were the basis of
pre-Civil War industrial development have become economically irrelevant because of the emergence of larger-scale iron mines in the West and in other countries. These deposits still exist, but it is unlikely that they will play an important role in economic production. This way of thinking suggests that we might best regard all resources as potentially producible, though some may be producible only at a very high cost. If this point of view is accurate, the model of land we are studying will be misleading.

As before, there are entrepreneurs who actually organize production by renting land and capital from capitalists and hiring labor. We suppose that labor is available at the conventional wage $w$. We denote the profit rate on capital by $\rho$, which is the amount of output the entrepreneur has to pay for the use of capital for one period.

The use of land as an input in production leads to the emergence of land rent. If entrepreneurs want to use all the land available in production, capitalists will be in a position as landowners to bargain for a rent on land, which we will call $r$. The payer of rent gets to use the land in production for one period. The dimensions of $r$ are $$/unit of land/year$. Since our land unit is the amount of land required to employ one dollar of capital, we could also express $r$ as $$/year, or $$/year, like the profit rate. If the profit rate is $10%/year and the rent on land is $5%/year, the entrepreneur has to pay $15%/year to rent capital and land from capitalists.

The profit an entrepreneur makes on each worker employed will be output per worker less the wage and rent on both capital and land. Remembering that we are measuring land in units so that $a = k$, the entrepreneurs’ profit is:

$$x - w - v_{kt}k - v_{rt}k$$

The entrepreneur must make zero profit, for the same reasons as in the one-input production model. If we reinterpret the profit share, $\pi$ to include rent on both land and capital, we can write this condition as:

$$v_{rt} + v_{rt} = \frac{1 - w}{k} = \pi$$

(11.1)

11.4 The Capitalist’s Decision Problem with Land

We will attack the analysis of this economy with the same methods we developed in Chapter 5. We will assume that there are a large number of identical capitalist wealth-holders each of whom begin owning the same share of the total capital and land in the economy. We continue to assume that these capitalists maximize the discounted sum of the logarithm of their consumption of output.

The introduction of land into the picture adds a new dimension to the typical capitalist’s decision. In the model of Chapter 5, the typical capitalist had to choose between holding capital and consuming at the end of each period. Now the typical capitalist has an additional choice: she has to choose how much of her wealth to invest in land and how much in capital.

Even though the amount of land in the whole economy is fixed, each individual capitalist could in principle own more or less land. Thus the asset price of land in terms of output (or capital), which we will call $p_{at}$, must adjust in each period to make the capitalists willing to hold the existing stocks of capital and land. The asset price of land is quite different from rent. The renter gets to use the land only for one period, while the purchaser of the land itself gets to keep the land until she wants to sell it, and collect rent on the land in all the periods she owns it.

The typical capitalist starts each period holding some land, $U_t$, and some capital, $K_t$.

At the beginning of period $t+1$, then, the typical capitalist’s source of funds will be the value of her depreciated capital together with the capital rent she has received, plus the value of her land together with the land rent she has received:

$$v_{kt}K_t + (1 - \delta)K_{t+1} + (p_{at+1} + v_{at})U_t = K_{t+1} + (v_{kt} - \delta)K_t + (p_{at+1} + v_{at})U_t$$

$$= (1 + v_{rt} - \delta)K_t + (p_{at+1} + v_{at})U_t$$

(11.2)

These funds must be divided between consumption, $C_t$, and holdings of capital and land in the next period, $K_{t+1}$ and $p_{at+1}U_{t+1}$. The typical capitalist’s budget constraint with land is thus:

$$K_{t+1} + p_{at+1}U_{t+1} + C_t \leq (1 + v_{rt} - \delta)K_t + [p_{at+1} + v_{at}]U_t$$

(11.3)

This constraint defines the typical capitalist’s utility maximization problem.

11.5 The Arbitrage Principle

The new element in the capitalist’s utility maximization problem is the decision as to how much of her wealth to invest in land and how much
choose \( \{C_t, K_{t+1}, U_{t+1}\}_{t=0}^\infty \geq 0 \) \hspace{1cm} (11.4)

so as to maximize \((1 - \beta) \sum_{t=0}^\infty \rho_t \ln(C_t)\)

subject to

\[ K_{t+1} + p_{at+1}U_{t+1} + C_t \leq (1 + r_t - \delta)K_t + (p_{at+1} + v_{at})U_t \] \hspace{1cm} (11.3)

\[ K_0, U_0, \{v_{at}, p_{at}, e_{at}\}_{t=0}^\infty \text{ given} \] \hspace{1cm} (11.5)

in capital in each period. We assume that the capitalist knows the paths of the price of land, the rental rate on land and the profit rate with certainty, which greatly simplifies this problem. In the real world the portfolio decision as to how to apportion wealth between competing assets, like equities and bonds, is highly dependent on the relative risk the wealth holder perceives in each choice. In our model, however, the issue of risk is absent. As a result the model's typical capitalist will choose between holding land and capital purely on the basis of which has the higher rate of return.

A capitalist who chooses to hold a unit of land during period \( t \) at the price \( p_{at} \) will have \( v_{at} + p_{at+1} \) at the end of the period, since she will collect the rent on the land and still have the land to sell. She could, alternatively, have invested the money in capital instead and had \((1 + r_t - \delta)p_{at}\) at the end of the period. These two returns must be equal if the capitalist is to be willing to hold both land and capital in her portfolio. This is the arbitrage principle, which plays a central role in modern financial theory. Rational wealth holders will hold assets with equal risk only if their anticipated rates of return are equal. In our model the two assets are capital and land. They have the same (zero) risk, so capitalists will hold both only if they have the same rate of return. Furthermore, the capitalist is indifferent as to how much of her wealth she holds in capital and land as long as the rates of return on the two assets are identical. The mathematical expression of the arbitrage principle is:

\[ 1 + r_t = 1 + v_{at} - \delta = \frac{p_{at+1} + v_{at}}{p_{at}} = 1 + g_{at} + \frac{v_{at}}{p_{at}} \] \hspace{1cm} (11.6)

The arbitrage principle tells us immediately that the rate of return to capital and land in each period must be equal, establishing a single rate of return, \( r_t \), that applies to both assets. The rate of return to capital is the rental to capital, \( v_{at} \), less the rate of depreciation, \( \delta \), and the rate of return to land is the rental to land, \( v_{at} \), plus the capital gain or loss on land due to the change in its price, \( g_{at} \).

The arbitrage principle reduces the capitalist's utility maximization problem with land to the same form as the utility maximization problem with one asset, capital, that we have solved in Chapter 5. To see this, define the capitalist's total wealth in each period, \( J_t = K_t + p_{at}U_t \). The arbitrage principle assures us that she will get the same rate of return, \( r_t \), whether she holds land or capital. Thus we can write the budget constraint as:

\[ J_{t+1} + C_t \leq (1 + r_t)J_t \] \hspace{1cm} (11.7)

But this is exactly the budget constraint in Chapter 5, with wealth, \( J_t \), substituted for capital, \( K_t \). This makes sense, because in the earlier model the only form of wealth was capital. We already know the solution to this problem: the typical capitalist consumes a fraction \( 1 - \beta \) of her wealth at the end of the period:

\[ C_t = (1 - \beta)(1 + r_t)J_t = (1 - \beta)(1 + r_t)(K_t + p_{at}U_t) \] \hspace{1cm} (11.8)

The Cambridge equation now applies to the growth of total wealth:

\[ J_{t+1} = \beta(1 + r_t)J_t \] \hspace{1cm} (11.9)

But the growth of capital itself is governed by the rule:

\[ K_{t+1} = J_{t+1} - p_{at+1}U_{t+1} = \beta(1 + r_t)J_t - p_{at+1}U_{t+1} \]

\[ = \beta(1 + r_t)(K_t + p_{at}U_t) - p_{at+1}U_{t+1} \] \hspace{1cm} (11.10)

The implications of capitalists' utility maximization in the model with land boil down to the arbitrage principle of equation (11.6) and the consumption function of equation (11.8).
11.6 Equilibrium Conditions

The analysis of Section 11.5 tells us how the typical capitalist will behave if she is confronted with a given path of prices, rents, and profit rates \( \{p_{t+1}, r_t, 1 + r_t\}_{t=0}^\infty \).

But the prices, rents and profit rates must be chosen so that the markets for capital, land rental and land owning clear in each period.

First consider the market for land as an asset. We have allowed the typical capitalist to make a free choice as to how much land she will own in each period. In equilibrium, however, she has to wind up owning her share of the actual amount of land in the economy \( U \). So land market clearing requires:

\[
U_t = U \quad (t = 0, 1, \ldots, \infty) \quad (11.11)
\]

But the rental land market has to clear as well. Entrepreneurs cannot plan to rent more land for production than exists. Furthermore, the rent on land will depend on whether entrepreneurs want to rent all the land or not. If there is so little capital in the economy that the entrepreneurs cannot use all the existing land, the land rent must be zero. If the land rent is positive, it must be the case that all the land is used. This turns out to be a key aspect of the growth path of this economy. Since we measure land in the same units as capital, rent will be zero if \( K_t < U \), and can be positive only when \( K_t = U \):

\[
K_t \leq U (= \text{ if } v_{at} > 0) \quad \text{or} \quad v_{at} = 0 \text{ if } K_t < U \quad (11.12)
\]

Thus we have two possible regimes in this economy. In the \textit{abundant land regime} there isn't enough capital to cultivate all the land, so some land will remain uncultivated, and land rent will be zero. As far as production goes, the abundant land economy is exactly like the Classical conventional wage model of Chapter 6.

But if capital grows to the level \( K^* = U \), the economy enters the \textit{scarc e land regime}. In this case the level of production is determined by the amount of land, not by the amount of capital.

\[
\begin{align*}
&v_{at} + v_{at} = \bar{p} \\
v_{at} = 0 \text{ if } K_t < U \\
r_t = v_{at} - \delta \\
p_{t+1} = (1 + r_t)p_{at} \\
J_{t+1} = \beta(1 + r_t)J_t \\
K_{t+1} = J_{t+1} - p_{at+1}U \quad (11.18)
\end{align*}
\]

From equation (11.18) we see that the capital gains from land can soak up some of capitalist saving, and thereby reduce investment in capital. This can be an important factor in the development of capitalist economies where a large proportion of wealth is in the form of land.

11.7 The Abundant Land Regime

We can work out the pattern of growth in the abundant land regime from the general equilibrium conditions.

In the abundant land regime \( K_t < U \), and the rent on land \( v_{at} = 0 \). The only way that land can compete with capital for a place in portfolios, as equation (11.16) shows, is for the price of land to be rising at the net profit rate. Thus in the abundant land regime:

\[
p_{at+1} = (1 + r_t)p_{at} \quad (11.19)
\]

Notice that the expectation of this price appreciation can justify a positive price for land, even though the rental on land is zero.

Now consider what is happening to the capital stock, by looking at equation (11.18). In the abundant land regime equation (11.19) holds, so the growth path of the capital stock follows the path:

\[
K_{t+1} = \beta(1 + r_t)K_t + \beta(1 + r_t)p_{at}U - p_{at+1}U \\
= \beta(1 + r_t)K_t - (1 - \beta)(1 + r_t)p_{at}U
\]
Equilibrium in the Abundant Land Regime

\[ v_{at} = 0 \]
\[ r_t = v_{at} - \delta = \pi \rho - \delta \]
\[ p_{at+1} = (1 + r_t) p_{at} \]
\[ K_{t+1} = (1 + r_t)(\beta K_t - (1 - \beta)p_{at} U) \]

If the price of land is low, there will be enough saving to allow the capital stock to grow.

In the abundant land regime both the price of land and the capital stock will rise, but as the price of land increases, capitalists will feel richer and richer and will consume a larger part of their resources, so that the growth of the capital stock will tend to slow down over time.

11.8 The Scarce Land Regime

Eventually the capital stock will grow to the point where \( K_t = U \), and the economy will shift to the scarce land regime.

In the scarce land regime output is limited by the availability of land, and there is no point in accumulating capital, since without more land extra capital will be worthless in production. Thus in the scarce land regime we know that:

\[ K_{t+1} = K_t = U = K^* \]

We also know that the net profit rate will be:

\[ r_t = v_{at} - \delta = \pi \rho - v_{at} - \delta \]

From equation (11.18) we can see that:

\[ K_{t+1} = \beta(1 + r_t) K_t + \beta(1 + r_t) p_{at} U - p_{at+1} U \]
\[ = K_t = K^* = U \]

Substituting for \( 1 + r_t \), and using equation (11.16), we have:

\[ K_{t+1} = K^* = \beta(1 + \pi \rho - \delta - v_{at})K^* + \beta(p_{at+1} + v_{at})U - p_{at+1} U \]
\[ = \beta(1 + \pi \rho - \delta)K^* + v_{at}(U - K^*) - (1 - \beta)p_{at+1} U \]

Since \( K^* = U \), the rents disappear from this expression, leaving:

\[ K^* = \beta(1 + \pi \rho - \delta)K^* - (1 - \beta)p_{at+1} U \]  \hspace{1cm} (11.20)

In the scarce land regime everything besides \( p_{at+1} \) in equation (11.20) is unchanging, so the price of land must be unchanging as well, at some level \( p^* \). Since the capital stock and price of land do not change from period to period in the scarce land regime, the wealth of the capitalists must not change either, so that:

\[ J_{t+1} = J_t = J^* = \beta(1 + r^*)J^* \]

The requirement that wealth be constant in the scarce land regime thus implies that the profit factor \( 1 + r^* \) is equal to the inverse of the capitalist saving propensity or utility discount factor, \( \beta \):

\[ 1 + r^* = \frac{1}{\beta} \]

Land rent, from equation (11.13), must satisfy:

\[ v^* = \pi \rho - \delta - r^* \]

From the land speculation condition, (11.16), we see that in the scarce land regime the price of land must be the present discounted value of future rents:

\[ p_a^* = \frac{v^*}{r^*} \]  \hspace{1cm} (11.21)

The scarce land regime is very much like Ricardo’s stationary state. The price of land is so high that capitalists consume all of their net income, and there is no growth. The profit rate after rent has fallen from its high level when land is abundant. Since the same capitalists own both land and capital, in this model there is saving and consumption out of both rents and profits, in contrast to Ricardo’s assumption that all rents were consumed and all profits accumulated. Thus in the scarce land regime the net profit rate can be positive, rather than falling to zero, as Ricardo predicted for the stationary state.
11.9 From the Abundant to the Scarce Land Regime

How does the economy that grows rapidly in the abundant land regime hook up with the stationary economy of the scarce land regime? The key is the initial price of land. As we have seen, for any initial price of land we can predict the paths of the price of land and the capital stock in the abundant land regime. On this path the price of land is going to rise to its stationary state level $p^*_n$, in some period. If the capital stock in that same period has grown to its stationary state level, $K^*$, the two regimes will fit together and the expectations of the capitalists will be exactly fulfilled. In the same period that the price of land rises high enough to stop the growth of capital, the capital stock will be just large enough to raise the rent on land above zero. The expectations of rising land prices will turn out to have been correct, and when land prices stop rising, rents will become positive to make land competitive with capital in capitalists' portfolios. This is a perfect forward equilibrium growth path. The price of land in the initial period is actually determined by the requirement that the two regimes fit together in this way.

If the price of land were too high in the first period, the capital stock would stop growing before it reached $K^*$ level, and rents would never become positive. The price of land and wealth would have to continue rising indefinitely, and eventually on such a path the capitalists would eat up all the capital in consumption. If the price of land were too low in the first period, the capital stock would reach $K^*$ while the price of land was still below $p^*_n$. Thus the capital stock would continue to grow, leading to unemployment of capital. Only when the market in the first period prices land as an asset at exactly the correct level will it be possible for the growth path to fulfill the expectations.

Example 11.1 Let $x = \$50,000/worker/year$, $\delta = 1/year$, $k = \$12,500/worker$, $w = \$20,000/worker/year$, and $\beta = .5$. $\rho = x/k = 4/year$, and $\bar{x} = (1 - (w/k)) = .6$. Suppose one hectare of land can employ $\$1,000 of capital and there are 1 million hectares of land available. The unit of land that can employ $\$1 of capital is 1/1000 hectare, so there are 1 billion units of land. Find the scarce land regime equilibrium, and the growth path leading to it starting two periods before.

Answer: In the scarce land regime we have

$K^* = U = \$1 billion$

$1 + r^* = \frac{1}{\beta} = 2/year$, so

$r^* = 1/year = 100\%/year$.

$v^*_n = \bar{x}p^* - (r^* + \delta) = \frac{(2.4 - 2)/year}{\$40/unit of land/year} = \frac{1,000,000 hectares}{\$40/unit of land}$

$p^*_n = \frac{\bar{x}}{r^*} = \frac{\$40}{1} = \$40/unit of land = \$400/hectare$

Thus in the scarce land regime the capital stock is worth $\$1 billion and the land is worth $\$4 billion. Suppose we take one step backward from the stationary state. In the abundant land regime we have

$v^*_n = 0$

$r = \bar{x}p^* - (r^* + \delta) = (6)(4)/year = 1.4/year = 140\%/year$

$1 + r)p^*_{n-1} = p^*_n$, so

$p^*_{n-1} = \$400/(2.4) = \$166.67/hectare$

$v^*_{n-1} = 0$

$K^* = (1 + r)(3K^* - (1 - \beta)p^*_n - U = \$1 billion = (2.4/year)(5K_{-1} - .5(1,000,000 hectares))$

and

$K_{-1} = \$1 billion$

In the period in which the capital stock reaches its maximum level, the land rent is still zero and the price of land continues to rise one more period before reaching the scarce land regime level.

If we take one more step backward, we have:

$(1 + r)p^*_{n-2} = p^*_{n-1}$, so

$p^*_{n-2} = \$166.67/(2.4) = \$69.44/hectare$

$v^*_{n-2} = 0$

$K_{-2} = (1 + r)(3K_{-2} - (1 - \beta)p^*_{n-2} - U = \$1 billion = (2.4/year)(5K_{-2} - .5(69.44)(1 million hectares))$

$K_{-2} = \$902.78 million$
Problem 11.2 Suppose in Ricardo's (see Problem 2.1) the production of 100 bushels of corn requires 1 acre of land, together with 20 bushels of seed corn and 1 worker-year. If there are 10,000 acres of land available, what is the maximum amount of seed corn capital that could be employed, and the maximum amount of corn output? If the wage rate is 20 lb/worker-year, what are the gross and net profit rates in the abundant land regime?

Problem 11.3 Find the land price, land rent, and gross and net profit rates in Ricardo (see Problem 11.3) in the scarce land regime when the wage is 20 lb/worker-year, and β = 4/5.

Problem 11.4 Make a spreadsheet program to calculate the growth path for Ricardo starting from the scarce land regime and working backward 20 years, calculating the asset price of land and the capital stock in each year.

11.10 Lessons of the Land-Limited Model

The model of growth with an absolutely limited resource like land underlines several fundamental insights of economic analysis.

A long-lived asset can have a positive price even when it yields no current return, like land in the abundant land regime, where the price of land is positive and rising even though the rent on land is zero. The price of land in this model is determined by forward-looking speculative forces. Capitalists will pay a positive price for land because they believe, correctly, that the rent on land will eventually become positive. Even before the rent becomes positive, landowners are rewarded at the average rate of return by the rising price of land.

Speculative pricing of assets is central to the operation of equity markets in capitalist economies. Equity claims on companies which pay no dividends and even have no earnings from which they might pay dividends can command a positive and rising price in speculative stock markets because asset-holders believe that the company may eventually become profitable. Even if the eventual profitability of a company is quite uncertain, and there is a significant probability in the minds of investors that it will never become profitable, its equity can still command a positive price because investors believe there is some probability that it will earn profits in the future. This effect, which the land model explains in a highly simplified setting, is the source of the wealth-creating powers of speculative asset markets. Hopes and dreams can be turned into hard cash, as long as enough speculators are convinced of the possibility that they will come to pass.

In the abundant land regime, the capital gains on land come to absorb a larger and larger part of the saving of capitalists, until at the moment land becomes scarce, the wealth represented by land is so large that capitalists stop saving altogether. This effect also occurs in real economies where wealth in land is very great. Wealthholders may believe themselves to be so rich that they stop saving, and make no funds available for investment in capital. The resulting economic stagnation has been a problem in some developing countries.

As Ricardo divined, a capitalist economy facing an absolute land constraint eventually reaches a stationary state where accumulation of capital ceases. In the limited land model the profit rate in the stationary state remains positive, in contrast to Ricardo's differential rent model. But the profit rate falls to the point where capitalists choose to consume all their net income, leaving nothing left over to finance new investment.

Ricardo disliked the idea of the stationary state, since he thought the accumulation of capital was the chief source of social change and improvement. He saw two factors that might, at least temporarily in his view, delay the stationary state. The first was international trade, and Ricardo's analysis became the bible of British free-trade advocates in the middle years of the nineteenth century. The effect of free trade, in Ricardo's eyes, was to incorporate all (or, at least, more) of the whole world's land in the economy, and thus to moderate the effects of diminishing returns. The extensive margin, instead of being confined to the narrow and rocky islands of Britain, could migrate to the fertile and empty prairies of North America, the pampas of Argentina, or the savannas of east Africa.

The other factor Ricardo saw as delaying the stationary state was technical change, particularly land-augmenting technical change that would raise the marginal product schedule for capital and labor, and move the stationary state further away from the current extensive margin.

But Ricardo, like contemporary theorists of limited economic growth, viewed both free trade and technical change as only temporary stoppaps delaying, but not preventing, the arrival of the stationary state. He believed that the law of diminishing returns would sooner or later assert itself.

The world economy has seen dramatic increases in the scale and scope of foreign trade since Ricardo's time, and equally dramatic technical change in agricultural and other resource-intensive production. The stationary state seems no nearer today than it did when Ricardo wrote. But perhaps it equally seems no further off. The warnings of contemporary theorists of limited growth, who see human society threatened...
11.11 Suggested Readings

The seminal work on the theory of rent is generally acknowledged to be [Ricardo, 1951, Ch. 2, “On Rent”]. The modern formalization of the Ricardian treatment by Pasinetti [1974] is highly recommended. For an in-depth discussion of land-rent and modern developments of the theory in the Classical tradition, see [Kurz and Salvadori, 1996, Ch. 10]. For treatment of land (and natural resources in general) in the basic Solow-Swan model, consult Meade [1961].

Chapter 12

Exhaustible Resources

12.1 Growth with an Exhaustible Resource

Land can neither be created nor used up in the model in Chapter 11. It is either abundant and has no immediate effect on production, or scarce, in which case it is an absolute limit on production.

Another important aspect of resource limitation is that some resources are *exhaustible*, in the sense that they get used up in production, and cannot be renewed. For example, certain mineral and oil resources exist as a quantity of ore or oil in deposits under the ground. As they are mined or pumped out they disappear. Once used, they cannot be replaced. We can use the same modeling approach as we used in the case of land to understand the fundamental economics of growth with exhaustible resources. This analysis is called the *Hotelling model* after Harold Hotelling, the economist who first solved this problem.

Just as it might make more sense to view land as ultimately producible than as absolutely fixed, it might make more sense to view mineral and oil resources as renewable rather than as non-renewable. First of all, new reserves of ores and oil can always be found by exploration and prospecting. In reality we do not know for sure exactly how large the ultimate reserves are; in practice it is possible at a cost to find new reserves. Second, the exploitation of mineral and oil reserves depends on the mining and drilling technology available. At any time there are known reserves that are too costly to exploit with existing technology. If society is willing to pay higher costs, more of these reserves become available. Furthermore, the technology is always changing, thus lowering the costs of exploiting known reserves. Oil companies now routinely drill wells that would have been impossibly deep 50 years ago. Some shal-