Introduction to Cost–Benefit Analysis
Looking for Reasonable Shortcuts

Ginés de Rus

Professor of Cost–Benefit Analysis
University of Las Palmas de G.C., Spain
University Carlos III de Madrid, Spain

Edward Elgar
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Foreword

The technique of cost–benefit analysis may seem an obvious approach to the appraisal of projects and policies to practising economists, but to many others it is confusing at the least, and sometimes even absurd. Isn’t it based on the proposition that only money matters? How can we place money values on people’s lives? Surely creating jobs is a benefit not a cost?

Ginés de Rus has written a book that concentrates on explaining the philosophy of the cost–benefit analysis approach to appraisal. He does give quite a lot of technical detail, but the key messages are put across simply and with many examples, so the book should be accessible to all concerned with appraisal.

The book incorporates the latest thinking on issues such as the use of distributive weights, the treatment of risk and uncertainty and the importance of institutional arrangements in ensuring the proper use of the technique. These issues are also blended into the approach taken in the examples, rather than treated as optional extras, as in some texts. The examples themselves are also very much the centre of current debate – in what circumstances to build new high speed rail lines, the case for privatization of water supply.

I first met Ginés many years ago when he attended my lectures on cost–benefit analysis as part of our MA in Transport Economics. It is a great pleasure for me therefore now to write the Foreword for his book on the subject, and to be able to recommend it as a clear and up-to-date text that will be of great value to all who need to know about this technique.

C.A. Nash
ITS, The University of Leeds, UK
Preface

The aim of this book is to try to understand whether public decisions, like investing in high speed railway lines, privatizing a public enterprise or protecting a natural area, increase social welfare. It is written not only for economists but for anyone interested in the economic effects of projects and public policies, generally financed with public money.

People may be interested in cost–benefit analysis for different reasons. Some individuals have to decide which projects will be undertaken, others have to inform those who decide on the social merit of these projects, or perhaps they need stronger arguments to defend their legitimate interest group position, to be better informed for the next election or simply because they enjoy applied economics.

One does not need to be an economist to understand the basics of cost–benefit analysis, but some previous knowledge of economics helps. Introductory microeconomics is clearly an advantage, not only to benefit more from this book but also for the interpretation and better understanding of many aspects of the economy and everyday life.1

The book is non-technical and I hope the exposition is simple and easy to follow. Its coverage is not comprehensive but I also hope the key elements for understanding and applying cost–benefit analysis are adequately treated. Although this is not a theoretical book it is analytic in the sense that it tries to follow the logic of arguments using some basic models, which are made explicit in their assumptions, limitations and implications.

The field of cost–benefit analysis has experienced a tremendous change in the last decades but the main economic principles behind this tool for public decision making remain unchanged. Moreover, despite the development of new techniques for the economic valuation of non-marketed goods, the refinement of demand forecasting or dealing with uncertainty, present practitioners in the field share the same aspiration as their colleagues in the past: to reach a reasonable degree of confidence regarding the contribution of the project to social welfare.

I am indebted to many people in the writing of this book. First, my colleagues Jorge Valido, Aday Hernández, Enrique del Moral and Eduardo Dávila for their help during the production process of successive drafts. They not only helped with corrections but also providing
valuable feedback. The book began as course notes for undergraduate and postgraduate courses at the University of Las Palmas and the University Carlos III de Madrid. I wish to thank my students and many other participants in short courses on cost–benefit analysis who read my class notes and helped me to decide how to convert them into book form.

I have received very useful comments and invaluable advice on draft chapters from Per-Olov Johannson, of the Stockholm School of Economics, and Chris Nash, of the Institute for Transport Studies at the University of Leeds. I am in debt to them for their patience, generosity and encouragement.

Finally I wish to thank the staff at Edward Elgar for their help, encouragement and advice as I prepared the manuscript.

I am the only one responsible for any remaining errors.

Ginés de Rus
University of Las Palmas de G.C., Spain
University Carlos III de Madrid, Spain

NOTE

1. Excellent introductory textbooks are, for example: Frank and Bernanke (2003), Krugman and Wells (2004), Mankiw (2003) and Samuelson and Nordhaus (2004). For an unconventional and outstanding overview of economic reasoning see Landsburg (1993). For a rigorous theoretical framework of cost–benefit analysis see, for example, Drèze and Stern (1987) and Johansson (1993).
1. Introduction

Any project can be viewed as a perturbation of the economy from what it would have been had some other project been undertaken instead. To determine whether the project should be undertaken, we first need to look at the levels of consumption of all commodities by all the individuals at all dates under the two different situations. If all individuals are better off with the project than without it, then clearly it should be adopted (if we adopt an individualistic social welfare function). If all the individuals are worse off, then clearly it should not be adopted. If some individuals are better off and others are worse off, whether we should adopt it or not depends critically on how we weight the gains and losses of different individuals. Although this is obviously the ‘correct’ procedure to follow in evaluating projects, it is not a practical one; the problem of benefit–cost analysis is simply whether we can find reasonable shortcuts.

(Joseph E. Stiglitz, 1982, p. 120)

1.1 THE RATIONALE OF COST–BENEFIT ANALYSIS

Cost–benefit analysis is not about money. It is not about inputs or outputs either. It is about welfare. Money is central to financial analysis but only instrumental in the economic appraisal of projects and policies. Money is the common unit in which economists express the social costs and benefits of projects. Volume of drinking water, accidents avoided, time savings and energy and labour consumed are measured in different units and we need a common unit of measure to express all these heterogeneous items in a homogeneous flow. This is the key role of money in cost–benefit analysis.

The creation of jobs is frequently presented as a benefit of a project, but labour is an input not an output. A motorway is not constructed to create jobs but to move people and goods. Workers building and maintaining a motorway represent a social cost equal to the net value lost in the next best use of this input. It is true that if a worker is unemployed, society does not lose as much as in the case of a similar worker already employed, but this only shows that cost values are context dependent.

The output of a project is easier to measure than its welfare effects. Public agencies report their activities with indicators such as passengers, water, electricity or the number of students taught within a training
programme, but cost–benefit analysis sees output as a means to increase welfare. The success of a new facility cannot be explained by the number of users. It is possible to subsidize prices to induce people to use the new facility without increasing social welfare. Therefore, cost–benefit analysis is interested in the social value achieved from the outputs of the project to compare with the value of other goods sacrificed elsewhere for the sake of the project.

Cost–benefit analysis is about the well-being of individuals affected by the project and not about the number of trips or visits. The change in welfare is what economists want to measure, and this is quite a challenging task because welfare cannot be directly measured. To solve this problem, economists have found an alternative: to use money as an expression of welfare. I do not know how great the utility of a particular individual is when driving his car from A to B at a particular date and time, but if I am able to determine the amount of money to charge for this trip that makes him indifferent between driving or not, then interesting things can be said. Cost–benefit analysis is not about money but money helps.

Cost–benefit analysis conceived as a toolkit for the selection of projects and policies, in the general interest of the society, presupposes the existence of a social planner, a benevolent government that compares benefits and costs before the implementation of projects and policies. Many economists and non-economists would consider such a view as naïve, to say the least. An alternative view explains a government’s action by the political power of different interest groups. Subsidies to agriculture, for example, could be better explained by the pressure of farmers than by an independent assessment of the social benefits and costs of agricultural policy.

Do we need to believe in the goodwill of the government to practise cost–benefit analysis? The answer is no. If we believe that a government’s acts are better explained by the influence of interest groups, cost–benefit analysis can show who benefits and who loses as a result of particular projects, and the magnitudes of the gains and losses. This assessment can be very helpful in explaining which policies are adopted or even in influencing a government’s decision. ‘Cost–benefit analysis may be in the battle against misleading information spread by self-interested political pressure groups. Still, these analysts can influence political outcomes by making enough voters aware of the true effects of different policies’ (Becker, 2001, p. 316).

To present the conceptual foundations and methods of cost–benefit analysis we will proceed ‘as if’ the government would aim for the best projects in the general interest of the society. Although we know of many cases that show that such an assumption is unrealistic, the simplification is harmless. As we proceed to identify benefits and costs, winners and losers
and try to measure and value the main effects of the project under evaluation, the analysis is not going to change whatever our particular beliefs on the government’s behaviour are.3

We have started assuming the existence of a benevolent government. This is not the only assumption and simplification in this book; in fact, there is no way to deal with the analysis of the economy but through the use of simplifying assumptions, replacing the actual world with a model that reflects the essence of the more complex reality that we want to understand.

To move forward, we need to clarify what is understood by acting in the general interest of the society. Let us consider that our benevolent government is evaluating the construction of a dam and a hydroelectric power station. The government doubts whether it should accept the project. By undertaking the project, the region would obtain electricity at a lower cost than without the project, recreation benefits, both in the stock of water (e.g. fishing and boating) and on the flow of reservoir release (e.g. rafting), and some jobs would be created at the time of its construction and during the lifetime of the project. Furthermore, there might be a multiplier effect, as the project would create new economic activity induced by the expenditure associated with the construction and operation of the project.

Economists point out that from the benefits described above we have to deduct some costs. First, the construction and maintenance costs, equal to the net benefits of alternative needs that have not been attended to because the public money has been assigned to the dam and power station, have to be deducted. They also argue that labour is an input, not an output, so it is a cost of the project, though its magnitude will depend on what is lost when the worker is employed within the project. The multiplier effect, if it exists, turns out to be irrelevant if it is also associated with the alternatives.

Second, all the other costs associated with the relocation of the inhabitants of the village in the area where the dam would be built and with the people negatively affected by the alteration of the flow and course of the river should be deducted. The magnitude of these costs could be substantial.

The government considers all the relevant benefits and costs regardless, in principle, of who the beneficiaries and the losers are (assume for simplicity that all the effects are inside the country) and the government decides to undertake the project if, given the available information, the society improves. Its decision is not based on the arguments of the private companies that will build the dam and power station, nor on the campaign of the opponents. The decision takes into account the whole society, with social welfare as the unique reference. The challenge for our
benevolent government is how to value all the benefits and costs and how to compare them given that beneficiaries and losers are individuals with different income, education, health, and so on, and are affected at different moments during the lifespan of the project.

This water project, as any other public infrastructure such as parks, high speed rail, highways, ports or the introduction of policies such as environmental regulations, can be interpreted as perturbations in the economy affecting the welfare of different individuals at different moments in time compared with the situation without the project or policy, which does not necessarily mean the status quo but what would have happened in the absence of the project or policy.

The assessment of the effects of the project requires a benchmark. It is necessary to compare the world with and without the project: to recreate an alternative world, or the so-called counterfactual. Cost–benefit analysis practitioners have to solve two main problems. First, they have to build the counterfactual and this means to replicate the world without the project, a dynamic world that evolves without the perturbation introduced by the project. This is not an easy task because the time period for this exercise may be quite long, 40 years or more, and the values of key variables will possibly change in each one of these years, only some of them in predictable ways. Second, the practitioner has to imagine the world with the project, forecasting the main changes with respect to the counterfactual that he has previously created.

The expected changes when the project is implemented are then the result of the comparison with the counterfactual: the worse the counterfactual, the better the project. Hence, it is important to present all the assumptions and the data used to complete this exercise. Transparency and ex post evaluation can help to avoid both innocent errors and strategic misrepresentation.

Suppose the counterfactual and the world with the project have been properly designed and the expected changes have been estimated: time savings, enhanced water quality or a reduction in the number of fatal accidents. Now, the analyst has to convert these values into monetary units ($) assuming that this is technically possible and morally acceptable.

We want to measure changes in the welfare of the individuals who compose the society; however individuals’ utility cannot be measured in the same way as the amount of electricity produced or the number of people displaced to build the dam. To decide on the goodness of the project we need to measure something that is unobservable. Furthermore what is observable – the production of electricity, number of individuals involved, extension of flooded surface, and so on – is not very useful if we do not translate the physical units into a common measure related to
changes in individual utility, which allows the comparison between what is gained and what is lost.

Hence, though the ideal way of measuring the impact of our project is through utility functions (we would measure the change in utility of each individual) the problem is that these utility functions and the associated utility changes are unobservable. Converting the unobservable utility changes through an ‘exchange rate’ between utility and income to observable monetary units gives us a way of calculating the impact of the project.

One alternative solution might be to submit the project to a referendum and to accept the outcome: that is, accepting the view of the majority. Let us have a look at this in more detail. Table 1.1 collects the information (expressed in monetary units) of the benefits and costs of those affected by the construction of the dam and the hydroelectric power station. Our society consists of five individuals.5

The individual B, for example, benefits from cheaper energy (+$2) but he also fishes downstream and the dam prevents him from practising his favourite sport in the initial conditions (−$8). The result is a net loss of $6 for individual B. We could interpret the values in the column ‘net benefits’ as the monetary compensation that will be needed with the project to leave each individual indifferent without the project: for example, the individual B would be willing to accept $6 and the individual A would be willing to pay $7.6

The column ‘net benefits’ allows us to anticipate that the project would be rejected in a referendum. Individuals A and D would vote in favour, but individuals B, C and E would vote against it. Would it be a good decision to reject the construction of the water project? To answer this question we have to check whether the construction of the dam is a social improvement and for this purpose we need to define a decision criterion.

A possible criterion is the strong Pareto improvement. To move from one situation to another is a social improvement (in the sense of Pareto) if at least one person is better off without making anyone else worse off.

<table>
<thead>
<tr>
<th>Individual</th>
<th>Benefits</th>
<th>Costs</th>
<th>Net benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>8</td>
<td>−6</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>4</td>
<td>−1</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>6</td>
<td>−5</td>
</tr>
</tbody>
</table>

Table 1.1 Benefits and costs of a hydroelectric power station (values in $)
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There are winners and nobody loses. We have seen that the referendum would result in the rejection of the project. Would it be possible, in these circumstances, to reach a Pareto improvement despite the outcome of the ballot?

Although it seems clear that the project under discussion would not be approved in a referendum, the society may gain from the project if, as it happens to be in this case, the benefits ($22) outweigh the costs ($19). Suppose the project is carried out and part of the benefits is used to compensate individuals B, C and E, so that their net benefit is zero, leaving them indifferent. Table 1.1 shows that, after compensation, there is a net benefit of $3 to share out as deemed appropriate. If the project is rejected this net gain would be lost.

On the other hand, when comparing the benefits and costs of the project, the magnitude of gains and losses counts. Individual C is against the project because it costs him $1, while D stands for the project because he gains benefits of $8. If we ignore the intensity of preferences, like in a referendum, we lose the potential gains arising from the project.

As we have seen, the Pareto improvement criterion requires no losers (i.e. there is full compensation to those initially harmed by the project). This rarely happens in the real world since, in many cases, the situation is similar to that described, but without full compensation to the losers. If a project produces a positive balance of benefits to the society as a whole and there are losers who, for some reason, cannot be fully compensated, it is normal practice to undertake such a project (the winners could have compensated the losers and still remain winners).

This criterion, in which the compensation is only hypothetical, is known as the potential compensation criterion, or Kaldor–Hicks criterion. If the losers are compensated, it would result in a Pareto improvement. Unless the project has unacceptable distributional consequences, the economic evaluation of projects and policies rests basically on the criterion of potential compensation just described.

To be more precise, we need – at least for a small project – to weigh individual (or group) \( i \)'s monetary gain with the marginal utility of income of individual \( i \), and with the social welfare weight attributed to individual \( i \) reflecting the social welfare function. Hence, the marginal social utility of income (see Chapters 2 and 11) attributed to \( i \) depends on what social welfare function we assume and the income distribution.

We can multiply the social marginal utility of income by the monetary valuation of the project (willingness to pay or willingness to accept) of individual \( i \) and sum over all individuals. So if initial welfare distribution is optimal (and where the marginal utility of income might vary across individuals since they might have very different utility functions: one
being a dedicated wildlife person, another being a passionate consumer of diamonds) the marginal social utility of income is the same for everyone. Then a project so small that it leaves the welfare distribution unchanged as a linear approximation can be evaluated simply by summing willingness to pay or willingness to accept across individuals/groups.

The common justification of the Kaldor–Hicks compensation criterion in practice is based on the argument that redistribution can be performed more efficiently through the fiscal system and that, overall, given the large amount of different projects being carried out, the positive and negative distributional effects tend to offset each other, and everybody wins in the long run, or they are unimportant, or the costs of identifying winners and losers and paying compensation are higher than the benefits.

1.2 STEPS OF COST–BENEFIT ANALYSIS AND OVERVIEW OF THE BOOK

The economic appraisal of investment projects and public policies must be flexible enough to capture the specific characteristics of each case study; however there exist some steps that have to be followed independently of the particular aspects of the project under evaluation. These are described below.

(i) Objective of the Project and Examination of the Relevant Alternatives

Before evaluating the project, its objective – that is, the problem to be solved – has to be clearly defined and the relevant alternatives identified. To analyse an isolated project without considering its role within the programme or policy where it belongs can lead to wrong conclusions. Moreover, before working with data and applying the methodology of economic evaluation, it is essential to analyse the relevant alternatives that allow the achievement of the same objective. An improper analysis of available alternatives can lead to important errors despite the methods and techniques being rigorous.

There are two a priori approaches for the practitioners in the appraisal of a project: first, when the analyst has to evaluate a particular project, for example, a price reduction in a public service; second, when the project is the improvement of a public service. If the goal of the regulator were to benefit consumers without damaging service quality, a possible measure could be to reduce prices, keeping the financial equilibrium with public subsidies. However there are also other policies to achieve this goal. An alternative could consist of introducing a system of incentives that
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compensates for efforts in the reduction of costs that allow price cuts. Another policy could be a private concession of the public service.

The consideration of different projects to achieve the same goal is a previous stage to the identification and quantification of benefits and costs in the evaluation given that the omission of more efficient alternatives is to lose the opportunity to gain better results. It is not enough to have positive social benefits, it is required that those benefits are greater than the benefits in the best available alternative. The same happens with investment projects. The question ‘is the investment the best way of solving the problem?’ must be answered. Other possible reversible and less costly options must be analysed, such as different management of the facility.

In the stage of the search for relevant alternatives it is very useful for the economist to interact with and receive feedback from other specialists more familiar with the technology or field related to the project. The objective of this step is to avoid errors because of a lack of precise information about more efficient methods to achieve the same goal. The greater refinement in the evaluation methodology would be useless if better alternatives had not been taken into consideration.

Finally, it is not convenient to define projects with too broad a scope because a positive evaluation of the aggregate can hide separable projects with negative expected returns. Therefore their inclusion, without differentiation, in a programme or a more global project can lead to wrong conclusions. To establish the limits of a project is not always easy but a careful discussion of the project with experts can allow us to distinguish intrinsic complexity from the inclusion of independent projects that are perfectly separable.

On the other hand it is nonsense to evaluate a project narrowly defined in the sense that its existence is not possible without complementary actions. Suppose that an investment project is composed of two main parts (e.g. a port and an access road) and the social net present value ($NPV$) is negative. The strategy of promoters could be to evaluate only the first part (the construction of the port) and, once it has been built, to present another project consisting of the complementary infrastructure to connect the port with the road network. In this case, the road project will probably be socially worthy because the investment cost of the already existing port is now irrelevant in the evaluation of the construction of the road, a principle that is not applicable to the lost benefits if the port cannot be operated.

In this chapter, we present the basic concepts of cost–benefit analysis and make explicit the simplification used to deal with the economic evaluation of projects that have medium and long term effects.
(ii) Identification of Costs and Benefits

Once the project is defined and bounded, it is necessary to identify the benefits and costs derived from its implementation. In some cases, this step is immediate and must not bring up greater difficulties, for example when projects have only direct effects (Chapter 2).

The identification of the costs and benefits of a project with significant indirect effects on other markets is more complicated because the impact must be located in markets different from the ones where direct effects are produced. The more reasonable approximation, when the analysis is not conducted within a general equilibrium framework, consists of identifying the main secondary markets affected by the project as would be the case with the evaluation of a new railway line that reduces the demand for an existing airport (Chapter 3).

In financial analysis, the identification is much simpler: benefits are revenues and costs are the payment of inputs valued at market prices. However, in economic analysis, benefits are those that are enjoyed by the individual independently of their conversion into revenues, and costs are net social benefits lost in the best available alternative.

Moreover, it is necessary to decide ‘who stands’ in cost–benefit analysis. Generally, country frontiers delimitate who must be included. Citizenship is the reference when the project has no global or controversial effects beyond the national boundaries. Sometimes it depends on who finances the project. In a co-financed project with supranational funds it would not be acceptable to exclude citizens of countries that contribute with their taxes to financing the project. On the contrary, it is not uncommon for a region only to consider local benefits and costs, ignoring positive or negative effects that take place in the rest of the country.

(iii) Measurement of Costs and Benefits

Projects’ benefits can be measured through individuals’ willingness to pay (or willingness to accept). Sometimes a monetary measure of the utility change that is derived from the project can be obtained by observing the behaviour of consumers in the market; that is, from market data. This is the case with the measurement of direct benefits in the primary market affected by the project (Chapter 2), the indirect effects in the secondary markets related to the primary market (Chapter 3), and in the valuation of non-marketed goods when the analyst can find an ‘ally’ market where some useful information is revealed about the willingness to pay of the individuals (Chapters 5 and 6).

On other occasions, economists have to estimate project benefits by
asking people directly about their willingness to pay (stated preferences); this consists of asking individuals about monetary quantities that reflect the change in their welfare thanks to the project. This approximation is used for non-marketed goods like environmental impacts or safety changes (Chapter 6).

In general, projects and public policies that are subject to evaluation imply the use or saving of resources. The costs of a standard investment project can be classified as: construction, maintenance, labour, equipment and energy – costs that are measured from the quantity of the inputs valued by their respective prices. From an economic point of view, the cost of input use is the net social benefit lost in the next best alternative. Market prices will sometimes be a good approximation of the opportunity cost, but in other cases it will be necessary to introduce some correction in market prices to approximate the social opportunity cost of the inputs, and this is what are called shadow prices (Chapter 4).

(iv) Benefits and Costs Aggregation

Benefits and costs occur in different periods of time and affect different individuals. The aggregation requires homogeneity, but benefits and costs that occur in successive years or affect individuals with different social conditions are not homogeneous. If they are directly summed, the implicit weight associated with each benefit or cost is the unity: a unit of benefit is identical disregarding the year or the individual.

Many infrastructure projects have lifetime periods over 30 years. Moreover, in the case of public policies that modify educational or health programmes, introduce or eliminate taxes, and so on, the ex ante lifespan is practically infinite. To discount future benefits and costs is a process of homogenizing to allow comparison. The discounting is performed using a discount rate greater than zero. This implies that the value of the benefits and costs decreases with time. The basic idea consists of the fact that individuals generally give more value to present than to future consumption and, therefore, future units of consumption are counted with a lower present value (Chapters 7 and 8).

Project costs and benefits affect individuals’ utility. To go from net individual benefits to aggregate social benefits implies redistributive effects. If the society gives, for example, more weight to the income of poor people, then the benefits and costs of a project cannot be added without social weighting. The net social benefit of the project should ideally be obtained as the weighted sum of the individual net benefits (Chapters 2 and 11).
(v) Interpretation of Results and Decision Criteria

The aspiration of the practitioner of cost–benefit analysis is to obtain a figure that summarizes the flows of benefits and costs. This figure is the NPV of the project, and helps with the decision to accept–reject or to choose between a set of projects.

To obtain a unique figure is not always easy. There are positive or negative impacts that resist the reduction to a monetary figure as happens to be the case with some environmental impacts. There are situations in which it can be appropriate to make a qualitative description of some effects, and then to attach this information to the NPV obtained with the effects that can be unambiguously measured.

Decision criteria based on the discounted value of benefits and costs are straightforward. If the NPV is positive and the redistributive effects of the project are positive or unimportant, the project increases social welfare. If we have to compare projects it may be necessary to homogenize before choosing the project with the higher NPV (Chapters 7 and 8).

Results must be subject to a risk analysis with the objective of determining the sensitivity of the NPV to changes in key variables. Ideally, it is preferable to compute a probability distribution of NPV instead of obtaining a unique NPV figure. Risk analysis allows the decision maker to have some information about the likelihood of the feasible results. Risk analysis does not eliminate the risk of the project but makes more evident the actual risk of the project to the decision maker (Chapter 9).

(vi) Comparison of the Project with a Base Case

In cost–benefit analysis, it is important to avoid a comparison of the project with an irrelevant base case. For example, comparing with the situation before the project, the NPV can be high because it can be hiding the fact that without the project that situation does not remain constant. There are maintenance policies or a minimum renewal of equipment, and so on, that could be implemented without the project. Thus, in the evaluation of the construction of a high speed rail line, we have to compare with a counterfactual where the demand changes and the supply of conventional rail and competing modes also changes.10

The situation without the project is known as the base case. We have to distinguish between do nothing and do minimum, which consists of the minimum intervention that is foreseen without the project. The distinction can be made clearer with an example. Consider that the objective is to substitute the water pipes that supply water to the city, because of excessive water leaks. In this project the reasonable base case is a do minimum
because, without the project, there will be maintenance operations and selective actions to avoid greater damage. Now consider that the project consists of a maintenance plan instead of investing in a new network. In this case the base case would be a do nothing.

(vii) Economic Return and Financial Feasibility

Cost–benefit analysis compares the social benefits and costs in contrast with financial analysis, which uses revenues instead of social benefits and private costs instead of social costs. However it is very important for the analyst to have a report that not only includes the economic return of the project but also the financial result or commercial feasibility of the project.

It is perfectly possible for a project or public policy to generate social benefits that exceed social costs and, at the same time, to present a negative financial result. Let us consider, for example, the case of a reforesting policy that reduces land erosion and delivers new space for recreation. Moreover, the responsible public agency obtains some revenues from charging for parking close to the recreation area. It is likely that this project presents a positive social $NPV$ and a negative financial result. The analyst must present both results to the decision maker for two main reasons.

First, the real world is characterized by the presence of budget constraints; therefore it is really useful for the public agency to have information on the social net benefit of the project as well as on the proportion of costs that are covered by revenues. Second, many projects produce a wide range of $NPV$s as a function, for example, of the pricing policy applied. It is usual for projects that admit the possibility of charging users to present different possible combinations of social $NPV$ and financial $NPV$. For example, a road project can be evaluated as a free access road or as a toll road. If the second option is chosen, several possible price structures exist: it is possible to discriminate by time, vehicle type or use intensity. It is likely that social benefits diminish with the toll; however collected revenues can contribute to fixed costs. To report the different options available and their social and financial $NPV$ increases the usefulness of cost–benefit analysis.

NOTES

1. We use the terms ‘utility’ and ‘individual welfare’ as synonymous.
2. George Stigler, Gary Becker and Sam Peltzman are among the top economists promoting the ‘interest group competition model’.
3. Nevertheless, once the conventional cost–benefit analysis is completed, we need to address explicitly the institutional design and the possible conflicting objectives, and we attend to that in Chapter 10.

4. We use the dollar symbol, $, as representing an undefined monetary unit without any relation to its actual market value.

5. We assume the individual is the best judge of his own interest; hence we ignore problems derived from distorted preferences (see Adler and Posner, 2001).

6. We assume for simplicity here that willingness to pay and willingness to accept coincide. For a technical discussion on the why these monetary measures of utility changes differ, see Chapter 11.

7. As is the case with legal expropriation.

8. There is a simplification here with the Kaldor–Hicks hypothetical compensation. Boadway pointed out that a positive sum of compensating variations, for example, is not equivalent to gainers being able to compensate losers. The problem known as the Boadway paradox is that the act of compensation might affect relative prices and hence welfare/utility of an individual (see Boadway, 1974; Jones, 2002).

9. For a Utilitarian society the marginal social utility or welfare weight is unity for all $i$, for a Rawlsian society it is equal to zero for all but the worst-off individual or group.

10. The comparison with an irrelevant alternative can be an error of the analyst, or a strategy for getting the project through.
2. The economic evaluation of social benefits

We like the cost–benefit criterion first because we think its application makes almost everybody better off over the long haul, and second because it is easy to apply. In other words, the benefits are high and the costs are low. The reasoning may be slightly circular, but the cost–benefit criterion recommends itself highly.

(Steven E. Landsburg, 1993, p. 105)

2.1 INTRODUCTION

The measurement of changes in social welfare and the decision criteria for the economic assessment of projects can be approached by modelling the economy as a set of individuals who, given their preferences, try to maximize their utility subject to two constraints: the available resources and the technology. A project changes the equilibrium in the markets in which individuals participate as consumers, owners of production factors, taxpayers, or affected by externalities, and cost–benefit analysis tries to measure the change in the utility of individuals to assess whether the project represents an improvement for an aggregate called society.

Given the scarcity of resources, people are typically forced to choose between different uses, and the available technology limits the quantity, variety and quality of goods produced from those resources. In a society where what matters is the welfare of its individuals, the focus is not on increasing production but the utility of individuals. In this sense, preferences limit the utility that individuals can reach for a given endowment of resources and technology, which is particularly relevant in the decisions that the public sector takes outside the discipline of the market.

In addition to the decisions of the individuals, the government also intervenes with projects that alter the equilibrium of some markets and affect social welfare. Its decisions affect the quantity, quality and composition of goods and services and their distribution. This chapter presents some economic concepts to build a basic framework for practical cost–benefit analysis.
The economic evaluation of social benefits

The basic model is described in section 2.2, where we simplify the society into several groups: consumer, taxpayers, the owners of production factors and the ‘rest of society’ to account for the external effects. Consumer surplus and producer surplus, key concepts in cost–benefit analysis, are explained in section 2.3, where the crucial distinction between price, cost and value is addressed.

To decide whether a project should be approved we have to compare its social benefits with its social costs, previously identified and measured. We need, first, to quantify in monetary terms the changes in individuals’ utility; second, to use some criteria for the aggregation of the benefits and costs affecting different individuals at different points in time; and, finally, to use some decision criteria to accept–reject or select from among a set of them.

In this chapter we focus attention mainly on the benefits, although we refer to the costs when it helps the argument (the detailed treatment of costs is in Chapter 4). One of the main points in this chapter is the distinction of two alternative procedures for assessing the net benefits in section 2.4: the sum of the changes in surpluses of individuals and the sum of the changes in the willingness to pay and in real resources, ignoring transfers. This distinction turns out to be quite important in practical cost–benefit analysis, and following carefully one approach or the other can help to avoid common mistakes.

Efficiency is not the only reference in society and who wins and who loses with a project are very important in public decision making. Section 2.5 discusses the conversion of income into individual utility and of individual utility into social welfare.

The evaluation procedures contained in this chapter apply to projects that are ‘small’. This requirement is necessary in order to simplify, to focus on the project’s impact mainly on the primary market and on the most directly affected secondary markets. The construction of a waste water plant is a project whose effects on inflation and government deficit are not generally expected to be significant. ‘Large’ projects that affect the macroeconomic variables should be analysed, where possible, with general equilibrium models.

When not specified, we will deal with market demand functions. Although the ideal would be to work with compensated demands, the results obtained using the observed quantities in the market are somewhat acceptable in many cases (see Chapter 11). In this book we assume that the conditions required for using market demands for monetary measurements of changes in the utility of the individual hold.
2.2 THE BASIC FRAMEWORK

A Simplified Economy

A project can be contemplated as a perturbation in the economy that affects the utility of individuals. The project’s impact occurs in the primary market (the directly affected market), and also in the so-called secondary markets, related to the primary because their products are complements or substitutes (see Chapter 3). Furthermore, the effects generally last for a long period of time, particularly in civil engineering projects, in the case of environmental impacts or policies that modify market structure.

The construction of a water irrigation project or a transport infrastructure, investment in education and the improvement of air quality have in common the effect on the welfare of the individuals affected by the project. Some individuals are better off and others worse off; they are located somewhere and this position is relevant to take into account whether their change in welfare counts or whether it is ignored. This is the question of standing, the problem of who counts in cost–benefit analysis. Usually the nation establishes the boundaries to who counts but sometimes it is the type of project, as is the case with the preservation of worldwide endangered species. On other occasions the co-financing by supranational organizations determines the countries that should be included. Although common sense can help in many cases, the difficulties could be significant in some projects with global externalities. To make the problem manageable, we will simplify as much as we can and work with a very simple model of the economy as described below.

In our simplified economy, in which we are going to evaluate projects, there are three owners of production factors: first, the ‘owners of capital’ ($O$), generally called producers, who have a variety of equipment, infrastructure and facilities where goods and services are produced; second, the ‘owners of labour’ ($L$) including employees of different skills and productivity levels, but in our simplified model they all belong to the only existing group of owners of labour or ‘workers’; and, finally, the owner of the third factor of production, the land.

The factor of production ‘land’ is defined in a wide sense including not only the soil for agriculture or the land for residential or productive uses, but also natural and environmental resources such as climate, water, air, flora and fauna and landscapes, which may be affected by projects. One important distinction is between the land under private ownership and the natural resources that are common property. We distinguish the ‘land owners’ ($R$) from another group of ‘owners’ of natural and environmental resources (also called ‘natural capital’), which we call the ‘rest of society’
where we also include any other external effect, like those affecting safety.

The combination of production factors creates a flow of goods and services. The income paid to those factors of production allows us to identify additional agents: ‘consumers’ (C) and ‘taxpayers’ (G), so that society is composed of six groups of individuals. Obviously, one can be a member of more than one group and must necessarily belong to group C. Moreover, this simplified society has another feature: the value of a unit of benefit (or cost) is the same regardless of to whom it may accrue (this assumption will be relaxed in section 2.5).

In this unsophisticated world, the social surplus (SS) is the sum of individual surplus:

$$SS = CS + GS + OS + LS + RS + ES,$$  \hspace{1cm} (2.1)

where consumers’ surplus (CS) is the difference between what consumers are willing to pay for the goods and what they actually pay. Taxpayers’ surplus (GS) equals tax revenues less public expenditure. The surpluses of workers (LS) and land owners (RS) are equal to the wage and land income, respectively, less the minimum pay they are willing to accept for the use of the factor; that is, its opportunity cost. Interestingly payment for land, like any other fixed factor, is usually what producers are willing to pay for the activity that requires the use of that fixed factor in competitive markets.

The surplus of the owners of capital (OS) is equal to firm revenues less variable costs and, finally, the surplus of the ‘rest of society’ (ES) includes the social value of non-marketed goods such as landscape, clean air and climate, and also external effects affecting health and safety levels that may change when a project is carried out, for example, a power plant that contributes to global warming or an investment in alternative energy sources that reduces it. There are many ways in which environmental goods can affect individual welfare, directly affecting individual utility, for example having access to a water reservoir of improved quality for recreation purposes or, indirectly, through the impact of this increase in water quality on the production costs of firms (see Chapters 5 and 6).

The government’s decision to invest in a project or to introduce a specific policy to change prices, costs or quality, affects the surplus of the social groups over the years for which the project continues producing its positive and negative effects, so that the total impact of the project can be expressed as the aggregate of changes in the surpluses of the individuals:

$$\Delta SS = \sum_{t=0}^{T} \delta^t (\Delta CS_t + \Delta GS_t + \Delta OS_t + \Delta LS_t + \Delta RS_t + \Delta ES_t),$$  \hspace{1cm} (2.2)
where \( \Delta \) denotes an increase or decrease in the surplus of the various agents and \( \delta' \) is the discount factor that allows us to express in present value the flow of benefits and costs over the lifespan of the project. For example, if individuals are indifferent between $95 in the present (year zero) and $100 within a year, the discount factor that converts the benefit in year one into units comparable to the benefits in year zero is equal to 0.95 (see Chapters 7 and 8).

To make the model even simpler, we assume that owners of factors of production (except the owners of capital) receive a payment equal to their opportunity cost, so their surpluses are equal to zero (they are indifferent between the situation with and without the project). We also assume that there are no taxes or public expenditure and there are no external effects and fixed factors. Finally, calling ‘producer surplus’ \((PS)\) to the capital owners’ surplus \((OS)\), the expression (2.2) is reduced to the familiar expression:\(^5\)

\[
\Delta SS = \sum_{t=0}^{T} \delta'(\Delta CS_t + \Delta OS_t).
\]  

(2.3)

To illustrate the change in social surplus with the underlying assumptions in expression (2.3), let us consider the case of a regulated public service operated by a private concessionaire as shown in Figure 2.1, whose total annual costs have an independent component of the volume of service, fixed costs \((K)\), and variable costs \((wL)\), where \(L\) is the quantity of labour and \(w\) the wage. Under these conditions, the average variable cost\(^6\) \((c)\) shown in Figure 2.1 is equal to \(wL/x\), where \(x\) is the number of users. For simplicity, assume a single period.

The demand \(D\) has a negative slope, indicating that if the price \(p\) goes down (up), the number of users of the service \(x\) increases (decreases). The super indexes 0 and 1 represent the situation without and with the project, respectively.

The government plans to reduce the regulated price from \(p^0\) to \(p^1\). Let us look at the expected change in social surplus. Without the project the initial price is \(p^0\) and the demand equals \(x^0\). The total revenue is represented by the area \(p^0ax^0\) and the total cost by \(cdx^0\) plus the fixed cost \(K\). Therefore the net profits of the firm providing the public service are represented by \(p^0ad0\) minus \(K\).

Consider the change in social surplus with the project. The increase in consumer surplus is equal to \(p^0abc\) and the reduction in producer surplus is represented by \(p^0adc\). The change in social surplus is therefore equal to \(abd\). If the government compensates the producer for the loss of surplus with the project, the change in social surplus \((abd)\) remains constant because the change in producer surplus is equal to zero, and the change
The economic evaluation of social benefits

in taxpayers’ surplus ($\Delta GS$) is negative and equal to the amount of the compensation ($p^0 adc$). The change in consumer surplus is unaffected. Therefore the change in social surplus is still represented by the area $abd$ once the change in taxpayers’ surplus is included. Income transfers do not change the results.

What about workers’ surplus? Looking at what happens in Figure 2.1 (right), the increase in production causes a shift in labour demand from $DL^0$ to $DL^1$, generating an increase in the level of employment $L^1 - L^0$. Should we count the earnings of the additional workers as an increase in the social surplus? If the answer was yes, we would have to add $efL^1L^0$ to the area $abd$ in Figure 2.1 (left).

However the answer is no, because while these additional workers receive a wage that they did not receive previously, they now lose the net value of what they were doing before they were employed, their opportunity cost (perhaps the value of leisure) that in the conditions represented by the labour supply $SL$ is exactly equal to the wages being paid ($efL^1L^0$). In the described circumstances, workers are indifferent between working and not working at the wage $w$, and therefore the change in their surplus ($\Delta LS$) is zero.  

2.3 PRIVATE AND SOCIAL BENEFITS

Price, Cost and Value

When a private company operating in a competitive market assesses whether a particular activity is profitable it compares the revenues and

![Figure 2.1 Price reduction in a public service](image-url)
costs attributable to such an activity. It can be argued that, in general, the value per unit of product for the firm matches the net price of the product sold, and the unit cost is equal to the internal cost to the firm in terms of the use of materials, labour and other inputs. However price is not necessarily equal to value, private value does not necessarily coincide with social value and private cost is not necessarily equal to social cost.

An open access natural park has a cost of maintenance and management and, although nothing is charged for its enjoyment, its use has value for the individuals who visit the park. The distinction between price, value and cost, and between private and social in the last two concepts, facilitates the task of valuing goods in order to evaluate projects.

Although the maximum that an individual is willing to pay for a good that can be purchased in the market reflects the economic value, expressed in monetary terms, of the good for the individual, the maximum willingness to pay does not necessarily coincide with what is actually paid: the user of a sport centre may be willing to pay an amount of money higher than the existing fee for the use of the facilities. The user of a health care service with free access may be willing to pay a certain amount of money for the use of this service, were the access conditional on the payment of a fee.

The private value of the good to an individual coincides with the social value of the good unless there are external costs or benefits. Continuing with the example of the health service, a vaccination campaign is beneficial not only to the users of the service but also to the society as a whole given the reduction in the spread of the disease. This is what is called a positive externality.

We must distinguish between total and marginal value. The first concept refers to what an individual would be willing to pay for the consumption of all units, while the second refers to the value of the last unit consumed. The case of water illustrates the difference between the two concepts: the total value is very high for any individual, while the marginal value of the last unit consumed is usually very low, at least in societies without scarcity problems. In a first approximation, the total value for the society is obtained by adding the willingness to pay of the individuals (for distributional issues, see section 2.5).

The price of a good is what is charged in the market for its consumption. If there is no price discrimination, the price does not coincide with the valuation of the individuals of any unit consumed but the last one, whose marginal value does coincide with the price because, with decreasing demand curves, for the remainder of the units consumed, individuals are willing to pay per unit more than the market price. The fact that the market price equals only the marginal value of the last unit exchanged in
The economic evaluation of social benefits

When a single price is charged, the total value or social benefit of the good is therefore higher than total revenues.

The cost of a good is the benefit lost in the next best alternative; that is, the net value of other goods that one must renounce for that good (the opportunity cost). In competitive markets, not distorted by subsidies or taxes for example, the price of the production factors used in producing the good is the opportunity cost of that good. The distinction between private cost and social cost is necessary in cases where the production requires not only inputs traded in the market, but non-marketed goods (e.g. environmental resources), or when it generates positive or negative externalities that the firm does not internalize; that is, that fall on the ‘rest of society’.

The difference between these concepts can be illustrated by the following example: a water supply firm serves a city, incurring a total cost that has two components: a fixed cost for the entire system of production and distribution ($K$) and a constant variable cost per unit of water produced and distributed ($c$). Figure 2.2 shows the water demand of the company and how at price $p_b$, consumers demand quantity $x_b$. It also shows how each unit of water has a different valuation.

The first units of water are possibly used to meet basic needs, so that their valuation is very high. The maximum reservation price ($\gamma$), at which the company would not sell any water, might be the price of an alternative supply system, for example, delivering water in a road tanker. As water consumption increases, the value of each additional unit decreases, indicating that the following units of water are intended to cover less valuable needs.

![Figure 2.2 Value, price and cost](image-url)
Every unit of water to the left of $x_b$ is valued above the price $p_b$, hence the user valuation of a unit of water only matches the price in the case of the last unit of the quantity $x_b$. As the variable cost of supplying one unit of water is equal to $c$, the difference between revenue, social benefits and costs is evident. The value of use of the water consumed, equal to the sum of the values of each unit, is represented by the distance between the horizontal axis and the demand ($\ldots + a + \ldots + b + \ldots + e + \ldots$). With a price equal to $p_b$, the sum of all the unit values equals the area between 0 and $x_b$ below the market demand $p = p(x)$; that is, the area $\gamma b x_b 0$. The revenue $(p_b b x_b 0)$ is part of the total value, and the private cost is equal to $K_0 c x_b$.

To calculate the social benefit of a policy that changes the consumption of water, it is generally incorrect to use revenues as social benefits since we have seen that revenues are only part of them. Observe that, at the right of $x_b$ in Figure 2.2, there are consumers whose marginal valuations are above the cost (for example at point $d$). When the price is equal to marginal cost ($c$), the marginal value coincides with the average variable cost and the price, but fixed costs ($K$) are not covered. Beyond point $e$, individuals value the units of the good below their marginal cost. Producing at the right of $x_e$ is inefficient since the value of the good is below its opportunity cost.

### Consumer Surplus and Producer Surplus in a Competitive Market

Figure 2.3 shows a competitive market in equilibrium. Firms in the market are price-takers and there are no barriers to entry or exit. The market demand $x_d = x(p_d)$ is the horizontal sum of individual demand and collects information on the amounts that consumers are willing to buy at different prices. The inverse of this function, $p_d = p(x_d)$, represents what consumers are willing to pay for different units of the good. This function approximately reflects the subjective valuations of the good and, in the absence of externalities, other distortions and problems of equity, the gross social benefit of exchanging different quantities of that good.

In Figure 2.3, consumers are willing to pay for the quantity $x_m$ the area $\gamma_d x_m 0$. If the price is $p_0$, the maximum they are willing to pay is represented by the area $\gamma_d b x_0 0$. It can be seen that the price chosen determines the total value of the good for the individuals. In the case of Figure 2.3, consumers pay $p_0$, consume $x_0$ and the total value of the good to them is $\gamma_d b x_0 0$. Since they pay $p_0 b x_0 0$, they obtain a surplus $\gamma_d b p_0$ that is equal to the difference between what they are willing to pay and what they actually pay. This is the concept of consumer surplus (CS), which is equal to the difference between what individuals are willing to pay ($WTP$) and what they actually pay (revenue represented by the price multiplied by the quantity). For the quantity $x_0$ and the corresponding price $p_0(x_0)$:
The economic evaluation of social benefits

The supply curve \( x_s = x(p_s) \) represents what producers are willing to offer at different prices. It is given by the sum of the horizontal supply functions of all the firms operating in the market. The inverse of this function \( p_s = p(x_s) \) represents the marginal cost of production. Between 0 and \( x_0 \) the total cost of production (assuming that there are no other variable costs than those represented by the supply curve) is represented by the area under the supply function between 0 and \( x_0 \). This area \( \gamma_s x_0 \) represents the opportunity cost of producing the quantity \( x_0 \). Producers sell \( x_0 \) at a price \( p_0 \), obtaining a gross profit equal to the area \( p_0 x_0 - \gamma_s x_0 \). The producer surplus (PS), which corresponds to \( x_0 \), is equal to the area \( p_0 x_0 - \gamma_s \) and it can be expressed as:

\[
PS(x_0) = p_0 x_0 - C_0, \quad (2.5)
\]

where \( C_0 \) is the variable cost of producing \( x_0 \).

With the assumptions made in the previous section, the social surplus is conventionally defined as the sum of the surpluses of consumers and producers, so we can express it as:

\[
SS(x_0) = CS(x_0) + PS(x_0) = WTP(x_0) - C_0, \quad (2.6)
\]

where the term \( p_0 x_0 \) in (2.4) and (2.5) nets out since it is an expenditure for consumers and a revenue for producers.

Expression (2.6) is represented in Figure 2.4 showing the net social surplus in a competitive market.
Introduction to cost–benefit analysis

surplus that is generated in the market represented by the shaded area in Figure 2.3. In the absence of distortions, the area represented in Figure 2.4 is the maximum surplus that the society can gain from this market given the preferences of consumers and the level of income and technology. Therefore, any distortion, such as the introduction of a tax or the presence of market power, reduces the maximum surplus. This reduction is the social cost of the distortion.12

Although Figure 2.4 looks like a demand function, it represents the net social value of each unit consumed. Note that the difference between what consumers are willing to pay per unit and the opportunity cost of these units is what Figure 2.4 represents. The relationship with Figure 2.3 is the following: the first unit exchanged in Figure 2.3 provides a social surplus equal to the difference $g_d - g_s$. In Figure 2.4 this difference is the intercept at the origin. When supply equals demand, $x_0$ in Figure 2.3, society is indifferent between adding an extra unit of production or not, as the net social benefit of this last unit is zero, as shown in Figure 2.4. If we go beyond $x_0$ then the (marginal) surplus becomes negative. That is why $x_0$ is optimal. In Chapter 3 we use this argument to justify not accounting for the indirect effects of the projects when these effects are only marginal changes in competitive markets.

The previous analysis is also useful for seeing the two main effects of the introduction of a tax: income transfer and efficiency loss. Assuming that the administrative costs of tax collection are zero, the introduction of a unitary tax ($\tau$) in the previous competitive market can be seen in Figure 2.5, which shows the cost in terms of the surplus lost ($bx_0x_1$) that society incurs to raise tax revenue ($\tau bx_10$). This illustrates what is called the deadweight loss of a tax and why project investment costs financed by taxes are sometimes multiplied by some conversion factors to reflect

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**Figure 2.4  Social surplus**

---
their opportunity costs. In Figure 2.5 investment costs represented by an income transfer equal to $\tau bx_10$ have an additional social cost of $bx_0x_1$.

We now assume that the tax is zero and analyse the social cost of negative externalities with the help of Figure 2.6. The consumption and production of goods do not only generate benefits and costs for the individuals who consume or produce them. The supply of a good can produce positive or negative externalities on other individuals and firms (e.g. the producer that pollutes a river reduces the welfare of fishermen and walkers).

In this case, the sum of the changes in consumer surplus and producer
surplus overstates the social surplus. We have to add the change in the surplus of the ‘rest of society’ represented by ΔES in the expression (2.2) to take into account the loss of welfare for individuals affected by the negative externalities associated with the production of the good.

To represent the impact of a negative external effect that we assume to be constant per unity and equal to $\varphi$ on the social surplus in Figure 2.6, we only need to add the externality to the private unit cost. Given that the equilibrium quantity does not change (firms do not internalize the externality), the effect on the surplus can be represented by shifting the curve of the surplus in the amount of the externality.

The externality $\varphi$ has the effect of reducing the social surplus in the area $dx_0$. Note that by subtracting the loss to the initial area $dx_0$, the resulting surplus is equal to the difference in areas $(ex_10) - (x_1x_0b)$. From $x_1$ to $x_0$, the production of the good or service in this market has a negative impact on welfare because what consumers are willing to pay is below the social opportunity cost. Then, why is it produced? Because producers do not internalize the cost of the external effect, making their market decisions with a cost function that does not reflect the social opportunity cost (this is what is called a market failure).

2.4 ALTERNATIVE APPROACHES FOR THE MEASUREMENT OF SOCIAL BENEFITS

Real and Monetary Values

Cost–benefit analysis aims to compare the flow of benefits and costs over the lifetime of a project. The relevant concepts are the social benefits derived from the increase in individual utility and the opportunity cost of the resources. The unit of measure used to express these benefits and costs is irrelevant.

Data can be presented in current or constant terms. The result of the evaluation will not change because the variables are expressed in real or monetary terms. In general the flow of benefits and costs is usually expressed in constant units of the base year; that is, in real terms, ignoring inflation. We are not concerned with the evolution of nominal values, but with the use of resources and the flow of benefits associated with the project.

However, the analyst might work with data expressed in current units. Sometimes the variables are expressed in these units (reflecting real changes and inflation). This is the case in infrastructure projects where users pay for the use and the private sector is involved as a manager or concessionaire.
Whatever the reason for the flows being expressed in real or nominal terms, the only recommendation is to be consistent. If the data are expressed in current units of each year, we have to use a nominal interest rate. If the data are expressed in monetary units of the base year, the real discount rate is the right one.

It may happen that the prices of some items evolve over or below the general inflation. In this case we must account for the difference. For example, suppose that the expected annual inflation over the 20 years of project life is 2 per cent, and the expected changes in the price of a raw material used in significant quantities for the project amount to 5 per cent, reflecting inflation and opportunity cost. By expressing the variables in monetary units of the base year, we only correct the element of general inflation (2 per cent), hence allowing the price of the raw material to grow at a 3 per cent rate to reflect the change in the real value of the resources. Although this approach is the right one, for projects lasting more than 20 years, figuring out the evolution of prices for each element of costs and benefits is not always possible.

The relationship between the real and nominal NPV is:

\[
\sum_{t=0}^{T} \frac{B_t - C_t}{(1 + i)^t} = \sum_{t=0}^{T} \frac{(B_t - C_t)(1 + \psi)^t}{(1 + i_n)^t},
\]

where:

- \( B_t, C_t \): benefits and costs in real terms
- \( \psi \): inflation rate
- \( i \): real discount rate
- \( i_n \): nominal discount rate.

From (2.7):

\[
\frac{1}{1 + i} = \frac{1 + \psi}{1 + i_n}.
\]

Solving for \( i \) we obtain the formula to calculate the real discount rate:

\[
i = \frac{i_n - \psi}{1 + \psi}.
\]

Expression (2.9) is often approximated as \( i_n - \psi \). For a nominal rate of 6 per cent and inflation of 3 per cent, the real rate is 2.9 per cent according to (2.9) instead of 3 per cent, which is obtained directly by subtracting inflation from the nominal rate. We assume for simplicity that \( i \) and \( \psi \) are constant over time.
Introduction to cost–benefit analysis

Sum of Surpluses or Change in Willingness to Pay and Resources

Let us assume we have decided to measure the flow of benefits and costs in constant terms and at market prices. There is still another choice between two alternative approaches. The first is the sum of changes in the surpluses of the different social agents. The second is to calculate the changes in willingness to pay and in the resources, ignoring the transfers of income. Both methods, properly used, lead to the same result. One of the most common errors in cost–benefit analysis is the double counting of benefits, and one of the best remedies to prevent it is to be systematic once one of the two methods described above has been chosen.

The following example illustrates the application of both approaches. It is worth emphasizing the importance of not mixing the two. Consider the case of a competitive market, as depicted in Figure 2.7, in a country with full employment. The demand curve shows the annual quantities demanded at each price and the supply curve represents the annual quantities offered at each price. There are no other effects on the economy beyond those reflected in the market represented in the figure. Without the project, the exchanged quantity is equal to $x^0$, where the superscript zero denotes ‘without project’ and the price is equal to $p^0$. Note that with this price the quantity offered is equal to the quantity demanded, the market is in equilibrium, and the consumer surplus (area $\gamma p^0 b$) and the producer surplus (area $\rho b \gamma$) generate the maximum social surplus.

What kind of project could improve the society in the described circumstances? In Figure 2.7, without the project $p^0 = c^0$. Suppose that if the government spends public money ($I$) on a new technology, the marginal cost of production is reduced from $c^0$ to $c^1$ regardless of the quantity produced. If

![Figure 2.7 The effects of a project](image)
the government invests $I$ at zero cost to firms, it is then possible to produce in two ways, either with the costs represented by the original supply curve ($S_N$) or with the new supply curve ($S_I$). Assume that the two technologies are compatible. It can be seen that it is cheaper to produce with the new technology only from $x^N_1$.

To determine whether the project’s benefits outweigh its costs, we must compare the government investment ($I$) at the beginning of year 1 with the social benefits over the lifetime of the project ($T$), assuming that $B$ and $C$ are situated at the end of the year. If the condition (2.10) holds, the project is socially profitable:

$$\sum_{t=1}^{T} \frac{B_t - C_t}{(1 + i)^t} > I.$$  

(2.10)

The introduction of the new technology reduces the market price to $p^1$, as Figure 2.7 shows, which is the new marginal cost of production, and the annual quantity increases to $x^W_1$ as all the consumers who are willing to pay the opportunity cost $c^1$ find someone who sells. It is interesting to note that the new quantity exchanged ($x^W_1$) is not all from the new technology, as it is still more efficient to produce the quantity ($x^N_1$) with the previous technology.

Consider the change in the surpluses of different agents. Consumer surplus changes from the situation without the project (area $g_{dbp}$) to the situation with the project (area $g_{dp^1}$), so the increase in consumer surplus is equal to the area $p^0 bdp^1$. Producer surplus without the project equals area $p^0 b\gamma_s$, and with the project $p^1 f\gamma_s$, so there is a reduction equal to $p^0 bfp^1$. Adding the change in the surplus of producers and consumers, we have a net gain equal to the area $bdf$.

The change in consumer surplus can be expressed as:

$$(p^0 - p^1)x^0 + \frac{1}{2}(p^0 - p^1)(x^W_1 - x^0).$$  

(2.11)

Equation (2.11) shows the two components of the consumer surplus. The first term represents the surplus thanks to the reduction of the price affecting the initial quantity ($x_0$). The second term (the additional quantity demanded when the price decreases) is multiplied by 1/2 because the surplus of the first unit is the initial price and the surplus of the last unit is zero (the willingness to pay equals the price).

Operating in (2.11) we obtain what is known in practical cost–benefit analysis as the ‘rule of a half’:

$$\frac{1}{2}(p^0 - p^1)(x^0 + x^W_1).$$  

(2.12)
The change in the producer surplus is:

\[ p^1 x^1_W - p^0 x^0 - C^1 + C^0, \]  

(2.13)

where \( C^0 \) is the cost without the project and \( C^1 \) the cost with the project.

Adding expressions (2.12) and (2.13) we obtain the change in social surplus:

\[ \frac{1}{2} (p^0 - p^1) (x^1_W - x^1_N). \]  

(2.14)

Let us examine in more detail the origin of this benefit. Recall that the price fall has increased the surplus of consumers in the area \( p^0 bdp^1 \). However part \( (p^0 bfp^1) \) of that gain is a transfer from producers to consumers that leaves the social surplus unaffected. This does not happen with the benefit represented by area \( bdf \), which does not exist without the project. This benefit has two sources. The first is by increasing the quantity produced \( (x^1_W - x^0) \), which does not increase the surplus of the producers because it is priced at a marginal cost, but it increases the surplus of the consumers \( (bde) \), who are willing to pay \( bdx^1_W x^0 \) but only pay \( edx^1_W x^0 \).

The second source of project benefits is the cost savings of producing the quantity \( x^0 - x^1_N \) that was previously produced at cost \( fbx^0 x^1_N \) and now is produced at cost \( fex^0 x^1_N \). The cost savings go entirely to the consumers since we are in a competitive market. To the change in the consumer and producer surpluses, during the lifespan of the project, we have to add the change in taxpayer surplus \(-I\).

Alternatively, the second approach is concentrated on the changes in the willingness to pay and resources, ignoring transfers. The increase in willingness to pay \( (area \ bdx^1_W x^0) \) comes from the additional units sold in the market, and the change in resources equals the incremental cost of the new production \( (area \ edx^1_W x^0) \) and the savings of producing \( x^0 - x^1_N \) with the new technology \( (area \ fbe) \). The annual gross benefit is represented by area \( bdf \). Once we have summed the discounted annual benefits during the lifetime of the project, we must subtract the investment costs \( I \).

Assuming that the annual net social benefit (denoted \( \overline{B} \)) is constant over time and equal to area \( bdf \) in Figure 2.7 and that \( T \) goes to infinity then the \( NPV \) is:

\[ NPV = \sum_{i=1}^{T} \frac{B_i - C_i}{(1 + i)^i} - I = \overline{B}/i - I, \]  

(2.15)

when \( \overline{B}i > I \), the \( NPV \) is positive and the project is socially desirable.

The case represented in Figure 2.7 can equally be applied to a public
policy. Suppose, for example, that the market in Figure 2.7 corresponds to an economy without international trade, initially in equilibrium with price $p^0$ and production (completely national) equal to $x^0$. The evaluated public policy consists of opening the economy and allowing imports at a lower price ($p^1$). Since imports are allowed, demand increases for the same reason as in the case of the investment project in new technology discussed previously. The only difference between the two cases is that the $NPV$ is greater now than in the case of the introduction of a new technology. The net social benefit of opening the economy to external trade is equal to $\frac{B}{i}$. In fact, openness to international trade is like the introduction of a new technology with zero investment costs. We can produce more cheaply and allocate resources (initially used to produce $x_0$) to more productive activities. From $x_1$ it is cheaper to import than to produce in the country. If the opportunity cost of the resources is represented by $S_N$, this policy is like the introduction of a new costless technology that allows an increase in welfare.

The Treatment of Taxes

We can apply the two alternative approaches described above in the presence of taxes. With the sum of surpluses, taxpayers’ surplus should now be included. Through the change in willingness to pay and in real resources, we must ignore transfers and include the willingness to pay that corresponds to the additional units sold independently of the fact that this ‘new’ willingness to pay is partly transferred to taxpayers through the tax.

Let us see how the presence of taxes affects the ‘rule of a half’ by introducing a unit tax on production. The market price ($p_0$) is equal to the unit cost ($c_0$) plus the tax ($\tau$). The change with the project consists of a reduction in price, resulting from a reduction in the unit cost of production ($c_1 < c_0$), and the subsequent change in the quantity demanded ($x_1 > x_0$).

Without the project, $p_0 = c_0 + \tau$. With the project, the price ($p_1 = c_1 + \tau$) is below the initial price ($p_0$) with a difference equal to the unit cost ($c_0 - c_1$). The change in social surplus is equal to the change in consumer surplus, the change in producer surplus (equal to zero in this case) and the change in the surplus of the taxpayers.

If the calculation of the benefit is realized through the change in willingness to pay and the use of resources, we have a reduction in the cost of producing the initial quantity without the project and an increase in the willingness to pay minus the cost of the resources used in the new production.

Applying the rule of a half, the change in consumer surplus (CS) can be expressed as:
\[ \Delta CS = \frac{1}{2}(p^0 - p^1)(x^0 + x^1). \] (2.16)

As the price is equal to the unit cost plus the tax, and the tax does not change:

\[ \Delta CS = \frac{1}{2}(c^0 - c^1)(x^0 + x^1). \] (2.17)

The change in producer surplus (PS) is equal to the change in profits (equal to zero in this case):

\[ \Delta PS = (p^1 - c^1 - \tau)x^1 - (p^0 - c^0 - \tau)x^0. \] (2.18)

The change in taxpayers’ surplus (GS) is limited to the increase in tax revenue as a result of increasing the quantity sold (the unit tax remains unchanged):

\[ \Delta GS = \tau(x^1 - x^0). \] (2.19)

The change in social surplus is equal to:

\[ \Delta SS = \frac{1}{2}(c^0 - c^1)(x^0 + x^1) + \tau(x^1 - x^0). \] (2.20)

The previous argument holds provided that the additional demand comes from income increases and not from the diversion of demand from other existing activities. If the increase in the quantity demanded \((x^1 - x^0)\) has been diverted from other activities, it must be borne in mind that in these activities tax revenues will be reduced, so the surplus of the taxpayers is unaffected, unless the tax rate is different, in which case only the difference matters. This shows that applying general rules such as ‘taxes should be ignored because they are income transfers’ can sometimes be misleading. When the new quantity is not diverted from other activities or, if it is, these activities are taxed at a different rate, the indirect tax (wholly or partially) is a benefit in the same sense as profits or consumer surplus.

### 2.5 WINNERS AND LOSERS

Virtually all investment projects and public policies unevenly affect the different groups of individuals that compose the society. It is quite unlikely for a project to distribute its costs and benefits uniformly. There are many projects whose costs are borne entirely by taxpayers, while the benefits are concentrated in a particular group; for example the construction of a
park in a residential area benefits the land owners in this area, and visitors and those passing by. In other cases, such as the construction of a power plant for a region, larger groups benefit, but their negative external effects, through for example, emissions are primarily borne by the population living close to the plant.

Sometimes the benefits go beyond the limits of the target group, and the generated externality distributes its effects across a wider population. This is the case with an immunization campaign improving the general health of all people, including the unvaccinated, because of the reduction in the probability of infection.

A challenging task is to find out the final beneficiaries of the project. A water policy that reduces the cost of irrigation can benefit producers who now have a lower cost of production, or consumers of agricultural products if the market is competitive. However, if land is scarce, it is likely that the owners of the fixed factor are the beneficiaries because of changes in the price of the land.

Why should we worry about the distributional issues in cost–benefit analysis? The rationale is that if society attaches more value to the utility of \( A \) than that of \( B \), and the project distributes its net benefits so that \( A \) wins 100 and \( B \) loses 100 with the same increase in individuals’ utility, the government considers that this project is not a zero sum game. Furthermore, even assuming that a unit of utility to \( A \) is socially equal to \( B \), one unit of income does not produce the same increase in utility unless the marginal utility of income is the same for \( A \) and \( B \).

**Conversion of Income Changes into Welfare**

When conducting a cost–benefit analysis instead of a financial analysis, the practitioner is looking for the net benefit for the whole society, resulting from the aggregation of the changes in utility of winners and losers. The result would be a good measure of the change in social welfare arising from the implementation of the project, if it were possible to measure and compare these changes in utility.

According to the analysis performed in Chapter 11, the change in social welfare (\( \Delta W \)) from a variation in the quantities of the \( n \) goods consumed by the \( m \) individuals who form the society can be somewhat expressed as:

\[
\Delta W = \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta W}{\Delta U_i} \frac{\Delta U_i}{\Delta x_{ij}} \Delta x_{ij}.
\]  

(2.21)

Reading from right to left in equation (2.21), we can see the process through which an increase in the amount of good \( j \) consumed by individual
ultimately affects social welfare. First, the project increases the amount of good \( j \) for the individual \( i \), by the amount \( \Delta x_{ij} \). Its consumption increases the utility \( (U) \) of that individual to a greater or lesser extent depending on the value of \( \Delta U/\Delta x_{ij} \), which is the marginal utility of the good for the individual and whose value depends on the preferences of the individual with respect to the good delivered by the project and the amount of the endowment that the individual has in the present. By multiplying the amount of good \( j \) by the marginal utility of that good \( (\Delta x_{ij}(\Delta U/\Delta x_{ij})) \), it can be seen that we convert the physical units of the project into units of utility. Then we multiply by the social marginal utility \( (\Delta W/\Delta U) \); that is, the social value of a unit of individual utility, leading to the change in social welfare.

To make operational the measurement of social benefits in (2.21) and taking into account that individual utility is not observable, we replace \( \Delta U/\Delta x_{ij} \) with \( (\Delta U/\Delta M)p_j \), since we know\(^{18} \) that for the individual \( i \), to maximize his utility, \( \Delta U/\Delta x_{ij} = (\Delta U/\Delta M)p_j \), where \( \Delta U/\Delta M \) is the individual marginal utility of income, and (2.21) is then equivalent to:

\[
\Delta W = \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta W}{\Delta U_i} \frac{\Delta U_i}{\Delta M_i} p_j \Delta x_{ij}.
\]

Condition (2.22) shows how the change in goods consumed by the individuals as a result of the project originates a change in welfare. Reading from right to left, a small increase in goods is converted into monetary units multiplying for the willingness to pay (the price for small changes).\(^{19} \) It is not enough to estimate the changes in the quantities of goods consumed by individuals, we also need to know the variation in the utility when income changes \( (\Delta U/\Delta M) \) and how much the society values the improvement experienced by the individual \( (\Delta W/\Delta U). \)

Taking into account that \( (\Delta W/\Delta U) \) \( (\Delta U/\Delta M) \) is the social marginal utility of income for the individual \( i \) (which we call \( \beta_i \)), the expression (2.23) is a weighted sum of the benefits (positive or negative) that individuals receive:

\[
\Delta W = \sum_{j=1}^{n} \sum_{i=1}^{m} \beta_i p_j \Delta x_{ij}.
\]

The introduction of weights, to account for income differences, alters the net social benefit of the project (provided that the weights are different from one). By making the weights depend on the level of income or other criteria, we could obtain different values of \( \beta \), so that the greater the \( \beta_i \), the greater the weight that society attaches to the benefits and costs of individual \( i \).
The problem with this approach is that we do not know the values of $\beta_i$ and the result of the economic evaluation of projects will depend on the values we choose. The introduction of weights changes the result of the project since the benefits and costs of the individuals involved will change depending on the value of a parameter that is quite difficult to estimate. The question is: what does the decision maker gain when altering the efficiency result by introducing the weights according to the expression (2.23)?

**A Practical Approach**

It is probable that an unweighted $NPV$ jointly presented with a breakdown of benefits and costs by income levels, geographical areas and some other characteristic of interest can be more helpful to the decision maker than a single figure obtained by applying social weights.

Moreover, a socially unworthy project, in terms of efficiency, could be approved if the weights for certain individuals are sufficiently high, which makes no sense before comparing it with a direct transfer of income. If redistribution is the objective, it pays to select the most efficient method of distribution.

The use of the potential compensation criterion, implicit in the $NPV$ calculation, implies that the value of the social marginal utility of income is equal to one and therefore we simply need to compare benefits with costs in order to determine whether the project is socially desirable or not. Although omitting the distributional effects of the project simplifies the evaluation, we run the risk of obtaining misleading results.

However the redistributive effects are not always important enough for their inclusion in the analysis. Moreover many projects include imperfect compensation schemes (i.e. compensation in a legal expropriation) that, although imperfect, ameliorate the undesirable redistribution effects of the projects.

Many public policies (health, primary education) and investment projects (electricity, telecommunications) will probably benefit the majority of the population when the final effects are considered. A public investment that lowers the cost of telephone services in a competitive market ultimately benefits the final users, which can be practically the whole population in a developed economy. In assessing a policy of this nature, redistributive effects might be negligible.

Redistributive effects can be significant, but the cost of identifying the winners and losers and how much they win and lose can be high enough to consider a priori that the gains arising from the information on the distributional effects do not outweigh the costs of obtaining such information. This is especially true when those affected are many and heterogeneous.
In the practice of cost–benefit analysis, equity weights are rarely included, it being much more common that the public agency that evaluates or commissions the evaluation is interested in disaggregating the effects of the project in terms of the groups and geographic areas affected, and also in having an estimate of the jobs that will be created as a result of the project. However remember the risk of double counting if labour has already been priced at its opportunity cost in the calculation of the project benefits (see Chapter 4).

THINGS TO REMEMBER

- It is worth having a simplified model of the society and a clear idea of the main impacts of the project before measurement. Time devoted to thinking about the characteristics of the problem and to assessing whether its appraisal can be reasonably dealt with in that simplified framework is a prerequisite to the identification and measurement of benefits.

- Revenues are not synonymous with social benefit. Under some conditions the willingness to pay for the project is the social benefit, and revenues are part of this benefit. The rest is called consumer surplus and is a benefit of the same nature as the revenue. Free access facilities do not generate revenue but social benefits are there, in the hands of social agents.

- Two main approaches are available for measurement: the sum of changes in the surpluses of economic agents and the sum of changes in willingness to pay and in real resources, ignoring transfers. Both of them lead to the same result, but they should always be kept apart. Be careful with taxes. Taxes can commonly, but not always, be treated as income transfers.

- Cost–benefit analysis deals with real values. The change in real things is what really matters: saving lives, improving the quality of water, reducing the resources required to produce the same amount of food, etc. The unit to measure real values is not important, so the only warning is to be consistent. The net present value of the project is not going to change whether inflation is included or not.

- Identify winners and losers when feasible. The net present value of the project accompanied by a list of who benefits and who is harmed can be helpful for the decision maker. At least do not forget that the aggregate $NPV$ is an efficiency measure that gives the same value to a unit of benefit and a unit of cost, regardless of who the affected party is.
NOTES

1. We use the term ‘project’ for both projects and policies.
2. For a more formal treatment see Chapter 11.
3. One definition of a small project is that it is virtually infinitesimally small. Then one can draw on envelope theorems implying that secondary effects ‘net out’. For example, if we change an output price (ceteris paribus) we obtain the supply function (from the profit function) minus the demand function, both multiplied by the marginal utility of income if we have a single-individual (or Robinson Crusoe) economy. Other supplies and demand are affected by the price change but these effects net out. This is so at least in the absence of tax wedges. Alternatively, one might consider a perturbation as a linear approximation. The project is so small that higher order effects can be ignored. This produces the same general equilibrium cost–benefit rule.
4. Sometimes individuals might be altruists in the sense that they care about damage caused by their activities on human beings or other species, or on the environment in other countries. For example, suppose country A replaces hydropower electricity with country B’s electricity produced by coal-fired plants. A’s citizens might – but need not necessarily – include the negative environmental impact in their utility functions (see Chapters 5 and 6).
5. Despite the simplification, with the society composed of consumers and producers, an individual can be both a consumer and a shareholder. We assume that the individual is able to distinguish, when asked about his willingness to pay for a project that affects the price of a product (and company profits), the effect on his surplus as consumer and as producer, so by adding the change in the consumer surplus and the producer surplus we do not incur double counting. For the effect of this bias on stated preferences see Johansson (1993).
6. Using a cost function where the marginal cost is not constant does not change the analysis, as long as the payment of factors at the marginal cost still holds.
7. When the wage is higher than the social opportunity cost of labour, the increase in employment is an additional benefit (see Chapter 4).
8. The total value of environmental goods includes the value of that use, and also the non-use or passive use value. Passive use value reflects what individuals are willing to pay for the good being available now or in the future, or simply for the mere fact of its existence. The economic interpretation of these concepts and their measurement are discussed in Chapters 5 and 6.
9. We assume that the market demand for water coincides with the compensated demand (see Chapter 11).
10. Assuming the income effect is very small.
11. If the industry is in long term equilibrium, profits tend to zero. The surplus of the producer that appears on the figure could be the compensation of some factor that remains fixed in the long term, for example, land. Producers have zero profits, since firms would bid for the fixed factor. The price of land would rise to absorb the producer surplus. Therefore, to obtain the social surplus, we would have to add the surplus of the owners of the fixed factors. Both procedures lead to the same result if one does not count the surplus twice (which is called double counting, a common error in cost–benefit analysis).
12. Plus the additional costs of the chosen mechanism. For example, in the case of the tax, the time lost in making the statements, the payment to staff dedicated to tax collection, fraud prosecution, and so on.
13. Costs and benefits can be expressed at market prices or factor costs (subtracting indirect taxes and adding subsidies). It seems preferable to express them at market prices and then make the necessary corrections, for example, when the market prices do not reflect the opportunity cost of resources (see Chapter 4).
14. The rule of a half assumes that all other prices remain unchanged, otherwise the demand curve (as well as the supply curve) might shift. Moreover it assumes a linear demand curve.
15. For a benefit $\bar{B}$ constant to perpetuity, the discounted present value is equal to $\bar{B}i$ (see Chapter 7).
16. This oversimplifies the benefits of open trade as far as international trade has its own transaction costs.
17. This is only true when the new quantity is not paid with money deviated from other markets with an identical tax rate.
18. We assume that the solution is interior (see Chapter 11).
19. When the change affects prices $p_j \Delta x_j$ is calculated with "the rule of a half".
3. The economic evaluation of indirect effects

... any project is likely to have some perceptible effect on the demand and supply of goods produced by other industries, the main effects of this type being in the industries which supply the materials used by the project, and the industries which supply goods which are either complementary to or competitive with the project’s output. If, as a consequence of a project, changes occur in the output of an industry for which, at the margin, social benefits equal social costs, no adjustment need be made.  
(Arnold C. Harberger, 1965, p. 47)

3.1 INTRODUCTION

Calculating the social NPV of a project through changes in the primary market is inadequate according to many who argue that investment projects and public policies have significant impacts on the economy beyond the visible effects in the primary market. A project may imply the reduction of production costs or an increase in the quality of the environment. Besides these direct effects, it is argued that the project may boost the economic activity in other markets and bring an increase in labour productivity and changes in firm location, among other effects of a broader scope than the direct effects described in the previous chapter. These other effects are called indirect effects and wider economic benefits.

Does cost–benefit analysis include all the relevant economic benefits of investment projects or does it ignore some important indirect effects on the economy? Indirect effects are those induced by the project beyond the primary market. When building a wind farm that reduces the price of energy, the direct effect occurs in the electricity market, but it also affects the firms that build the windmills or supply inputs for the wind farm.

There are other indirect effects, since the product of the primary market, the supply of electricity, is related to other markets whose products are complements to or substitutes for the electricity produced by windmills: the electricity produced by a fuel burning plant is a substitute for the electricity from the windmills, and appliances are a complement (e.g. the demand for electric stoves increases if the price of electricity falls).
There is another relationship of the primary market with other markets that tends to be confused with the earlier one but whose nature is completely different, as we can see in section 3.3. The relationship is established when the product of the primary market affected by the project, such as electricity or transport, is an input for the production of other goods that use these as inputs. With respect to the goods for which the electricity is an input, the list is endless (including the entire production process of this book).

We therefore have a broad set of candidate markets for measuring the effects of the project beyond the primary market. In general, and this significantly simplifies the job of the analyst, the indirect effects can be ignored provided that, in the markets where they occur, the marginal social benefit equals the marginal social cost.

Therefore, in competitive markets in which there are no distortions such as externalities or taxes, the indirect effects are equal to zero. In the case of distortions we must include the indirect effects, but it should be noted that these indirect effects do not necessarily increase social benefits. Many projects are small enough to keep income constant, so if demand in a secondary market increases it must be decreased in some other markets. So it is basically an almost philosophical question of the magnitude of the discrepancy between price and marginal cost in the different markets. In any case it is very difficult to trace these effects unless the analyst has a computable general equilibrium model.

We deal with the analysis of the indirect effects in two parts. The first, which is covered in section 3.2, includes the effects that could be called genuinely indirect. The second, when the product of the primary market is an input for other markets, is analysed in section 3.3. In general, we assume that it is possible to identify some markets where the main indirect effects concentrate and then proceed as if the rest of indirect effects net out.

In general, the challenge in identifying the benefits in cost–benefit analysis is to separate the genuine indirect effects from those that are double counting. With this purpose, it is useful to distinguish, within the economic effects that are not direct, between the indirect effects and the wider economic benefits, covered in section 3.4, and to emphasize that both can increase or decrease the social benefit of the project. Finally, in section 3.5, we address the so-called territorial effects and regional development induced by infrastructure investment projects.

3.2 INDIRECT EFFECTS

Investment in a natural area used for recreation increases the demand for hotels and restaurants in a village near the place affected by the project.
This effect is one of the many indirect effects of the project. It does not occur in the primary market in which we have measured the willingness to pay of the individuals who use the new recreational area. These effects occur in the secondary markets and the question is whether or not they should be included in the calculation of the NPV of the project. The answer depends on the existence of distortions in these markets that make the price and the marginal costs differ.

Indirect effects to be included in calculating the economic profitability of the project are reflected in the following expression, assuming prices are left unchanged:

\[
\sum_{t=1}^{T} \sum_{j=1}^{n} \frac{\left(p_{jt} - c_{jt}\right)\left(x_{jt}^1 - x_{jt}^0\right)}{(1 + i)^t},
\]

where the first term in parentheses in the numerator is the difference between price and marginal cost in each secondary market \( j \) at a given period of time \( t \) and the second the increase or decrease in the quantity produced on the secondary market as a result of the change produced by the project in the primary market. The two sums indicate that these effects are accounted for in the \( n \) secondary markets in the economy over the \( T \) years of the project, discounted at the discount rate \( i \). Assuming that there is a single secondary market and the effect occurs only in the present, the expression (3.1) simplifies to:

\[
(p - c)\Delta x.
\]

It is important to emphasize that the indirect effects are sometimes a simple relocation of a previously existing economic activity and therefore the increase in the occupancy of hotel rooms (\( \Delta x > 0 \)) in a particular city should be accounted for (we will see immediately how) in exactly the same manner as the decline in hotel occupancy (\( \Delta x < 0 \)) that happens in another city, or any other activity, that without the project received the expenditure of those visitors.

When the marginal social benefit equals the marginal social cost (\( p = c \)), an increase or reduction in production in the secondary market, following the shift in demand with the project, does not change the social surplus. In competitive markets, without changes in prices, the indirect effects should not be included because the price equals the marginal cost of production. In these conditions, an increase or decrease in the quantity induced by the relations of complementarity or substitutability with the primary market does not change the social surplus since the willingness to pay for the increase in production is equal to its opportunity cost.
When there are distortions in the secondary markets \((p \neq c)\) and the products of these markets are complements to or substitutes for the product of the primary market, the indirect effect may increase or decrease the benefits of the project. The ultimate sign of these effects on the \(NPV\) of the project will depend on whether the price in the secondary market is higher or lower than the marginal social cost and the combination of one of these cases with the fact that goods are complements or substitutes.

The figures in this section are built, for simplicity, under the assumptions of demand shifts to the right in the secondary market \((\Delta x > 0\) in (3.2)). Depending on what happens in the primary market (rising or falling prices, changes in quality, etc.) and the relations of complementarity or substitutability with the secondary markets, demand shifts may be to the right or to the left. Assuming that price is equal to marginal cost in the primary market, we have several cases depending on whether the secondary market price is equal to marginal cost.\(^1\)

**Indirect Effects in Competitive Markets**

Suppose that the price of electricity falls, increasing the demand for electricity (primary market) and shifting the demand for electric stoves (secondary market). This indirect effect of reducing the price of electricity is represented in Figure 3.1.

The reduction in the price of electricity increases the demand for electric stoves in this secondary market, and demand changes from \(D^0\) to \(D^1\) in Figure 3.1, without changes in utility since, in the primary market, the willingness to pay for the electricity incorporates the utility of its use, and the price of electric stoves is unchanged because the supply is perfectly

![Figure 3.1](image_url)
elastic as assumed in Figure 3.1; or in the case of a positive slope supply, because the shift in demand is marginal in the market for electric stoves, leaving the price constant.

Let us see what happens with the producer’s surplus. In the market for electric stoves, the increase in demand induced by the price reduction of electricity does not change the price of the good in the secondary market (whose supply is perfectly elastic as Figure 3.1 shows), with incremental revenue equal to incremental costs (area $abx^1x^0$). Therefore the producer’s surplus remains unchanged in this market.

The case depicted in Figure 3.1 is the case of a complement of the good directly affected by the project. In the case of a substitute the analysis is similar, changing the direction of the demand shift, now to the left but with the same result: as the social surplus does not change in the secondary market we can also ignore the indirect effect as these changes do not affect welfare.

**Negative Externalities and Subsidies**

When in the secondary market the price is lower than the marginal social cost, we have either the case of a negative externality or the case of a subsidy (which does not internalize a positive externality), as shown in Figure 3.2.

Consider the case of a negative externality (atmospheric pollution, noise, etc.). Initially, we are located at the equilibrium point $e$ of a secondary market in which there is a constant negative externality equal to $\varphi$ per unit. While the increase in demand does not affect the price, the social cost is now higher than the private cost. In the initial equilibrium, the marginal social cost exceeds the price paid by users in $p_0$. With the increase in demand from $D^0$ to $D^1$, there is no change in the utility of consumers or

![Figure 3.2  Secondary market with externality or subsidy](image)
producers (revenue $edx^1x^0$ equals private cost), but there is an additional increase in social cost represented by the area $abde$, which reduces the surplus of the ‘rest of society’ and therefore must be included.

Interpreting $\varphi$ as a subsidy in Figure 3.2, the analysis is similar to the previous one. The marginal social cost is $p_0 + \varphi$ but the firm produces as if the cost is equal to $p_0$. When the demand shifts to the right in the secondary market, there is no change in consumer and producer surpluses; on the contrary, taxpayers’ surplus is reduced in the area $abde$.²

This analysis is valid provided there are no other distortions in the economy (or those other distorted markets are unaffected by the project under consideration); that is, if all other affected markets are of the type illustrated in Figure 3.1.

**Taxes and Unemployment**

In the earlier cases the increase in production in the secondary market reduced the social benefits of the project. In the cases of revenue raising taxes and involuntary unemployment the price is above the marginal cost in (3.2) and the indirect effects of increased production in the secondary market have a positive effect on the project’s $NPV$.

Consider the case of a unitary tax included in the market price in the secondary market as represented in Figure 3.3. The marginal cost is now $p_0 - \tau$ instead of $p_0$, and therefore the increase in the demand for electric stoves in the secondary market, induced by the reduction in the price of electricity in the primary market, produces an increase in the social surplus equal to the tax revenue $abde$.

With the demand shift the price does not change, but the social cost is now lower than the private cost. Initially the social marginal cost is equal to $p_0 - \tau$. Given the increase in demand from $D^0$ to $D^1$, with no change in

![Figure 3.3  Secondary market with taxes](image-url)
the utility of consumers or producers (revenue $abx^1x^0$ equals private costs), there is an increase in the surplus of taxpayers equal to the area $abde$. Taxes have to be accounted for in this case as an increase in the social benefit of the project. In this case, taxes are not a transfer of income to be ignored in order to avoid double counting (unless in another secondary market a similar reduction in tax collection occurs).

In the event that there is unemployment and the opportunity cost of labour is less than the market wage, $p_0 - \tau$ could be interpreted as the social opportunity cost of producing electric stoves in a situation of unemployment and minimum wage regulation, and $p_0$ as its market price. The area $abde$ should be counted as a social benefit (an increase in the surplus of workers) since, given the increase in demand in the secondary market, the real cost of the resources used to increase production is equal to $edx^1x^0$ (the opportunity cost of production). The area $abde$ would be measuring the benefit of job creation (see section 4.7 for a more detailed treatment).

### 3.3 DIRECT EFFECTS MEASURED WITH A DERIVED DEMAND

A shortcut to the measurement of the benefits of a project that reduces the cost of an input used by many firms is to calculate the benefits in the input’s market, instead of calculating firm by firm the surplus of different economic agents. When building a new infrastructure that reduces transport costs, the effects are enjoyed by firms whose profits may increase, or by consumers who travel at a lower generalized price or consume goods at a lower price.

Transport demand is derived from the needs in the markets for final goods and services. This is called *derived demand* and is very useful for measuring the direct effects of the reduction in transport costs on social surplus. An interesting result in the evaluation of investment projects that reduce transport costs is that we can focus our attention on the market in which the reduction in the cost of transport occurs and measure in this market the benefits of the project, without having to worry about changes in other markets (provided they are competitive) using the transport service.

By concentrating the effort on measuring the demand for transport we avoid the identification and measurement of changes in the surpluses of a large number of firms and their consumers. This does not mean that companies in other markets using the services of transport do not benefit from the project that reduces the cost of transport, or that consumers do not benefit from lower prices; it is simply to avoid counting the same effect...
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twice, because the benefits of reducing such a cost have been measured in the primary market for transport.

Figure 3.4 in its upper part shows the impact of the reduction of transport costs on the supply of a product and its effects on producer and consumer surpluses. $S^{0+}$ and $S^{0-}$ are the supply curves without the project, gross and net of transport cost respectively. The situation without the project shows a competitive market in equilibrium where we assume that producers are located at the same point in the space but consumers are located at different distances from producers, which offer the product according to the supply function $S^{0*}$. The supply ($S^{0*}$) and the demand determine the equilibrium without the project at price $p^0$ and the quantity produced $x^0$.

The market price paid by the consumers $p^0$ is not fully received by the producers since the cost of transport (assume price equals marginal cost in the transport market) reduces the price that producers receive ($p^{0-}$). All

Figure 3.4 Measurement of benefits with the derived demand
the units produced are transported to their ultimate consumers and therefore at the cost of transport without the project \( c^0 \) (equal to \( p^0 - p^{0^-} \)) the quantity produced is equal to that effectively transported.

So far this figure provides little help. However, when we assume a reduction in the cost of transport with the project, Figure 3.4 shows how the derived demand for transport is very useful for measuring welfare changes. With the project, the cost of transportation is reduced, and in the upper part of the figure the supply function is shifted to the right so that the new equilibrium is determined by the supply function \( S_1^* \) and the demand, being the new price with the project \( p^1 \) and the quantity produced \( x^1 \). In the lower part of the figure \( x^0 \) increases to \( x^1 \).

The change in social surplus, in the upper part of the figure, is composed of the change in the surplus of consumers and producers. Applying the rule of a half (see Chapter 2), the change in consumer surplus is equal to:

\[
\frac{1}{2} (p^0 - p^1) (x^0 + x^1), \tag{3.3}
\]

and in producer surplus:

\[
\frac{1}{2} (p^1 - p^{0^-}) (x^0 + x^1). \tag{3.4}
\]

These are the effects of the reduction in transport costs (under the assumption of perfect competition). The difficulty of measuring such surpluses in the markets for final goods is easy to imagine; but the demand for transport as represented in the lower part of Figure 3.4 shows a shortcut to the measurement of the change in social surplus by applying the rule of a half with the transportation costs and trips realized.

\[
\frac{1}{2} (c^0 - c^1) (x^0 + x^1). \tag{3.5}
\]

Expression (3.5) is equal to the sum of (3.3) and (3.4). Measuring the change in social surplus with the market for the transport input following (3.5), we can not add as benefits the impacts of reducing the cost of transport on the market for goods in those competitive markets using transport as an input. If we do so, we would double count the benefits.

### 3.4 WIDER ECONOMIC EFFECTS

We have distinguished between indirect effects and wider economic benefits. Indirect effects are those that occur in the secondary markets linked
to the primary market by relations of complementarity or substitutability and have already been analysed in section 3.2, concluding that they have to be included whenever there is a distortion in these markets and the cross elasticities are not zero. The impossibility of concluding a priori whether the final sign of the indirect effects would be positive or negative, depending on the circumstances of the affected markets, was also highlighted.

Market distortions produce other effects, the so-called *wider economic benefits*, which in principle should be included in the calculation of the social profitability of the project because they are not double counting and can be important in certain contexts. With no intention of being exhaustive we discuss some of these effects such as the agglomeration economies, imperfect competition in markets that use transport as an input and the effect of increasing competition as a result of the implementation of the project.

**Agglomeration Economies**

Agglomeration economies are in fact a positive externality that firms generate when they locate close to other firms. If productivity increases with the density of firms in an area, productivity depends on the location decision of each firm. A company, when deciding where to install the plant, takes into account its own benefits but not the increase in the profits of other firms.

The benefits of time savings of a transport project are valued by the firms, which now change the location and increase the density of firms in a city or an industrial park (as measured by the derived demand function), but would be lower than the increases in productivity enjoyed by all the firms. Following the same reasoning, the reduction in the density of firms in the area where the companies were initially located reduces productivity and therefore it is a negative effect that must be accounted for.

There are several reasons explaining why firms in areas with a higher firm density are more productive, and why firms choose these locations despite the higher labour and land costs among other drawbacks. These reasons include access to wider markets, the availability of a more specialized labour market that matches the needs of firms, and the access to technologies and the production processes of firms in the area.

There is empirical evidence of the positive effect of a higher *effective density* on productivity, understanding effective density as an indicator of the degree of agglomeration, such as employment in the area and its surrounding areas (weighted by the generalized cost of transport), so that it appears that an increase in effective density increases the average productivity of the area.
A project that reduces transport costs may also induce an increase in the concentration of jobs in an area where there are economies of agglomeration by reducing the cost of commuting for workers who, with the project, are now more willing to move to the city or the industrial park. However the opposite might also occur if the reduction in transport costs encourages the dispersion of economic activity. For an urban project that reduces the costs of travel within the city it is more likely that the positive effect will dominate, while for an intercity transport project the possibility that the dispersion will increase cannot be ruled out, depending on a set of local factors such as land prices, wage differences between areas, and so on (see Duranton and Puga, 2004; Graham, 2007; Venables, 2007).

Productivity gains arising from economies of agglomeration must include the additional tax revenues that are collected as a result of the increase in economic activity. Additional gains, for example that workers value when they take their decision to migrate to the area of highest density of employment, are net of taxes; nevertheless tax revenues are also productivity gains (Venables, 2007). This is another example, by the way, that taxes are sometimes mere transfers and at other times represent welfare gains.

**Imperfect Competition in Markets that Use Transport as an Input**

To estimate the welfare gains of a project that reduces the cost of transport, it is quite practical, as we saw earlier, to use the derived demand in order to avoid the difficulty of measuring the effects on every market affected by the project. A necessary condition for the final effects to be calculated in the transport market was that product markets were perfectly competitive.

What happens in the presence of market power?

In markets with imperfect competition the price is higher than the opportunity cost, and firms are in equilibrium at a point where consumers are willing to pay above the marginal cost. This is the typical textbook case of a monopoly. If the producer is unable to discriminate prices, a price reduction to attract new customers reduces revenues on the initial units and thus what the monopolist compares with the marginal cost when deciding to increase or decrease its output is the marginal revenue, not the price.

The economic consequence of a profit maximizing monopolist is well known: a suboptimal equilibrium quantity and the corresponding loss of efficiency. How does this fact affect the evaluation of a project that reduces the cost of transport for firms with market power? Figure 3.5 shows the effect of a reduction in transport cost on the marginal cost of a profit maximizing monopolistic firm whose unit cost changes from $c^0$ to $c^1$. It
can be seen that initially the firm is in equilibrium at point \( a \), and now the equilibrium is at point \( b \) with a reduction in consumer prices from \( p^0 \) to \( p^1 \). The increase in producer surplus is represented by the area \( c^0fhc^1 \). The consumer surplus increases in the area \( p^0abp^1 \).

One might think that in the transport market these areas are measured by the change in total surplus in Figure 3.4 by applying the rule of a half as reflected in expression (3.5); however, although this is the case for competitive markets, where the marginal change in output has zero value, this does not happen in markets with imperfect competition, where the price–marginal cost margin is positive.

To see this intuitively, we can think of a very small change in Figure 3.4, so the areas in the figure reduce to the following expression:

\[
-x \Delta c + (p - c) \Delta x. \tag{3.6}
\]

If the product market that uses transport as an input is competitive, the social value of the last unit exchanged is close to zero as the price is equal to the marginal cost (as shown in Figure 3.4) and therefore expression (3.6) is equivalent to (3.5), whereas in the imperfect competition case, the value of the last unit equals the price minus the marginal cost (greater than zero as shown in Figure 3.5) and therefore the application of (3.5) ignores the second term of expression (3.6). Although consumers are not willing to pay more than the equilibrium price for an additional unit and neither are producers willing to produce it (because its marginal revenue equals the marginal cost), there is an additional benefit by producing that...
The economic evaluation of indirect effects

Marginal unit that the transport project makes possible and it is equal to 

\( p - c \).

Ignoring transfers, the change in total surplus equals the cost reduction in the production of \( x^0 \) (area \( c^0fgc^1 \)) plus the willingness to pay (net of costs) of \( \Delta x \) (area \( abhg \)).

Besides the increase in social surplus, measured in the primary market, we now have an additional benefit not included in the primary market and that is a reduction in the deadweight loss, because of the existence of imperfect competition, equivalent to the increase in production multiplied by the difference between the price and the marginal cost (Venables and Gasoriek, 1999). It should be noted that the effect can be negative if in the secondary markets with market power firms sell less because the reduction in transport costs affects, for example, a product positively that is a substitute for the product in the secondary markets.

**Higher Competition**

When transport costs are high, projects that reduce them can facilitate the entry of new firms who find it profitable to offer their products compared with the situation without the project when the incumbent firms were protected by the barriers to entry that in fact represent the transport costs. This effect should not be confused with welfare gains arising from the expansion of output in imperfectly competitive markets when they reduce transport costs. We can say that this effect, previously discussed, does not increase competition but that firms with market power (e.g. in markets with product differentiation) find it profitable to increase their production given the reduction in transport costs.

The pure competition effect, with the entry of new firms, is less likely to occur in countries with a mature infrastructure network. In these countries significant increases in efficiency resulting from the fact that a transport project reduces travel time are not expected. However it is an effect that could be important in any project that affects a part of the country that is poorly connected and in which some firms enjoy market power because of poor accessibility.

**Recent Findings in the Literature on Wider Economic Benefits**

Recent research on the nature and magnitude of wider economic benefits is still far from producing practical rules for the economic evaluation of transport projects subject to the usual constraints of time, money and technical resources. Some recommendations are as follows (OECD, 2007). The additional
economic effects not captured in conventional cost–benefit analysis exist and they have their origin in the existence of increasing returns, agglomeration economies, market power or the benefits of a broader labour market. Moreover, firms and households take long term decisions in reacting to the changes in transport costs. However the sign and magnitude of these effects are very different between projects and it is not possible to transfer them to new projects being evaluated.

A practical suggestion for small projects is to work under the assumption that the wider economic benefits do not exist or are unimportant. Although this approach runs the risk of ignoring them in the case where they are significant, there is consensus on the fact that this risk is offset by the elimination of the risk of double counting and delays in project evaluation. For large projects or for the evaluation of investment programmes it may be justified to undertake more sophisticated analyses.

Aggregate studies that focus on global impacts have problems detecting the direction of causality. Moreover they do not contain information with a sufficient level of detail on the particular infrastructure that makes the results useful in evaluating projects. In addition, there is some confusion about whether these studies measure wider economic effects, ignored in standard cost–benefit analysis, or whether they only measure the final impact of the direct effects already measured.

More detailed ‘microscopic’ studies seeking to capture the effects of transport cost reductions on the internal reorganization of firms and families are scarce. This is not surprising since this type of longer term response is difficult to integrate into micro-focused modelling that analyses the interaction in the markets; however it is known that companies and households make reorganization decisions in response to new transport conditions.

Ex post studies are also scarce. Existing ones have found no robust evidence on the existence of wider economic benefits. The latest research suggests that if one wants to go beyond the conventional cost–benefit analysis to include any possible additional benefits, one should distinguish between direct benefits and impacts on productivity, competition and the labour market. Furthermore, when there are spatial spillover effects, regardless of including the wider benefits, we must expect different results in the evaluation if the size of the geographic area analysed varies. Moreover, the economies of agglomeration may also have negative effects because of the increase in traffic congestion that may even lead to negative wider economic benefits.

In the modelling of agglomeration effects, as with the spillover effects, there is concern about using a ‘black box’ of questionable utility in the evaluation of projects. Advancing in the ‘microscopic’ analysis of the
agglomeration benefits would be very useful for a better understanding of the effects on production, product distribution and access to inputs, and for a better understanding of mechanisms that tend to spread the activity because of the reduction in transport costs through just-in-time processes or the advantages of having separate plants to avoid upward pressures on wages.

The existence of spatial spillovers requires extreme care in estimating the effects of local agglomeration. For example, the study of the London Crossrail link concluded that the benefits of conventional cost–benefit analysis had to be increased by 20 per cent to include the effects of agglomeration, but it could not demonstrate the extent to which these wider benefits were losses in other geographical areas.

From the available empirical evidence and the evaluation of the experts on whether conventional cost–benefit analysis is sufficient to estimate the social profitability of a project, the general recommendation is to be extremely cautious since, although economists are advancing with the knowledge and measurement of wider effects, they are still far from turning the results into practical rules for their inclusion in cost–benefit analysis.

The risk of double counting is so high that the best approach is not to include wider economic benefits in small projects and focus the efforts on the direct effects. With the available evidence, it does not seem reasonable to transfer the results from other studies, using percentages or similar procedures, if we take into account the variability in the magnitude of the wider effects, and even the sign when the negative effects resulting from congestion outweigh the agglomeration benefits.

3.5 LOCATION EFFECTS AND REGIONAL DEVELOPMENT

Even assuming that there are no wider economic effects, and that the benefits of the project have been properly identified and measured, the location of these benefits is usually an important element for the decision taker, together with the net social benefit, when considering public investment decisions.

The location of firms and the induced increase in economic activity is one of the arguments used in the defence of investment projects in public infrastructure. It is assumed for example that the construction of highways or railway lines, which reduce transport costs from a poor region to another more developed region, will enable greater economic growth for the former thanks to the higher attraction of the poor region for the location of firms.
This argument is not supported by empirical evidence. Roads can be used to export goods from the poor region to the rich region but also from the rich to the poor. Therefore, in principle, a reduction in transport costs between the two regions does not guarantee the desired location effects. When one admits the existence of agglomeration economies, a reduction in transport costs can facilitate a greater concentration of activity in the rich region, which could now export its products at a lower cost to the poor region, rather than directly produce in the poor region.

The emphasis on the location effects of firms in deprived areas may be of interest to governments or lobby groups who want the project to be approved; however, from the point of view of the economy as a whole, it does not seem reasonable to introduce as benefits the more than doubtful location effects of firms that could even materialize in the opposite direction than the one originally planned.

The so-called new economic geography has shown that the effects of reducing transport costs in the less developed regions do not only depend on the characteristics of the project but also on the economic context. A simplified explanation of the difficulty of establishing a priori the location effect of an infrastructure is as follows.

Let us consider a country with two regions, the rich ($R$) and the poor ($P$), separated by an inadequate transport infrastructure (we will call the initial quality level – travel time, route conservation, safety, etc. – ‘bad’). There is only one factor of production, labour, and wages are initially identical in the two regions.

Economic activity and the location of firms between the region $R$ and the region $P$ are in equilibrium. There are more companies in $R$ and more unemployment in $P$. The equilibrium is explained by several factors including the level of infrastructure. First, producing in $R$ and exporting to $P$, using the infrastructure, has the advantage of reaping the economies of agglomeration (economies of scale and access to specialized inputs, for example) that give rise to a lower unit cost in $R$ than in $P$. Locating the firm in $P$ has the advantage of avoiding the transport costs incurred when producing in $R$ and exporting to $P$.

The trade-off between the pros and cons of producing in one region has been resolved with an initial level of activity and location of firms that we take as a starting point and that allows us to call one region ‘rich’ and the other one ‘poor’.

Suppose we have to evaluate a project that allows the infrastructure to pass from level ‘bad’ to ‘good’, reducing the transport costs between the two regions. How will this project affect the location of firms between the two regions? It is often argued that investment projects in infrastructure
improve the situation of the poor region by allowing its development. The empirical evidence is not so optimistic, with several possible outcomes.

In the simplified world described here, the availability of a better infrastructure changes the initial equilibrium. The only change that has occurred is a reduction in the transport cost, hence it is more profitable to produce in $R$, take advantage of economies of agglomeration and export to $P$ using the improved infrastructure. The result is the relocation of firms in $R$. The new economic activity raises wages in $R$ and attracts the labour force from $P$ to $R$, which allows the containment of wages and the reinforcement of the relocation in $R$.

Suppose now that a new project improves the infrastructure from level ‘good’ to ‘very good’ and that transport costs are further reduced. We can visualize two plausible scenarios. In the first, wage agreements are negotiated at the national level. If so the cheaper costs will intensify the relocation effect benefitting region $R$. The second scenario involves regional wage bargaining. In this case wages will rise in the region $R$ and fall or remain constant in $P$.

If the wage differential is sufficiently high and transport costs are sufficiently low, it may happen that companies move to the poor region and export to the rich. Everything will depend on the trade-off between the benefits of agglomeration economies in $R$ and the lower production costs in $P$.

The previous example is a note of caution on the difficulty of predicting the final location effects, or regional development effects, of infrastructure investments projects without incorporating other factors, sometimes more critical than the infrastructure per se, such as the labour market situation. Companies, when making their location decisions, take into account a set of factors, one of which is transport cost. A reduction in one factor changes the equilibrium and may foster agglomeration or dispersion, depending on the combined effects of the set of relevant factors.

**THINGS TO REMEMBER**

- Secondary markets are affected by the change in the primary market, the one where the direct effects of the project take place. Indirect effects and wider economic benefits may be significant.
- The measurement of direct benefits can sometimes be performed with an input derived demand, as is the case with a transport project. Derived demand has the advantage of concentrating valuable economic information that simplifies the calculus of the direct benefits.
Indirect effects may be positive or negative depending on the relationship of complementarity or substitutability between the goods in the primary and the secondary markets (cross-elasticities different from zero), and the existence of distortions.

Wider economic benefits can also be present because of, for example, economies of agglomeration that induce productivity increases when a project helps to increase the employment density in an area. Some of the positive effects derived from agglomeration economies have the side effect of congestion or the reduction of similar benefits in those areas losing firms and workers.

A reasonable line of action is to concentrate attention and effort on the identification and measurement of the direct effects, and the closest substitutes and complements in distorted markets, ignoring the minor adjustment in many markets where measurement would be too costly to justify the benefits in terms of accuracy in the calculation of the NPV.

Regional development is one of the arguments used in the defence of major infrastructure projects in poor regions, but this argument is not supported by empirical evidence. Transport infrastructure can be used in both ways and a reduction in transport costs between two regions does not guarantee the desired location effects.

NOTES

1. If price is not equal to marginal social cost in the primary market, the number of possible cases increases.
2. For simplicity we ignore the deadweight loss produced by the taxes required to subsidize the amount \( abde \) (see section 4.8).
4. Opportunity costs, market and shadow prices

Circle the best answer to the following question
You won a free ticket to see an Eric Clapton concert (which has no resale value). Bob Dylan is performing on the same night and is your next-best alternative activity. Tickets to see Dylan cost $40. On any given day, you would be willing to pay up to $50 to see Dylan. Assume there are no other costs of seeing either performer. Based on this information, what is the opportunity cost of seeing Eric Clapton?

A. $0; B. $10; C. $40; D. $50

4.1 INTRODUCTION

Policies and projects evaluated within the public sector are undertaken because their social benefits are expected to exceed their social costs (at least this would be desirable). It is difficult to find situations where there are benefits without any cost; in general, to obtain benefits, it is necessary to use production factors whose opportunity costs are usually greater than zero. In other words one must give up some goods in order to acquire others.

The correct measure of costs is essential to any economic assessment. The market price of the production factors employed in the implementation of any project does not always reflect the opportunity cost. The costs of projects can come from the use of land and natural resources, labour and capital. From an economic point of view, the cost of the input is the social benefit in the best available alternative, which has been lost in order to undertake the project.

Frequently, when the demand for an input for the project is small with respect to the market size of the input, its market price is a good approximation of the social cost resulting from its use. If the project involves a change in the market prices of some inputs (e.g. in the case of a specific input available in short supply), the economic valuation of this input requires us to distinguish between resources of new supply and resources diverted from other uses as a consequence of rising prices derived from the shift in input demand in the factor market.

In sections 4.2 to 4.4 we discuss some useful concepts for the identification
and assessment of project costs such as the opportunity cost, avoidable versus sunk costs or incremental versus average cost. In section 4.5 we analyse how costs change with and without the project. Then, in section 4.6, we show the circumstances in which the market prices of inputs are reasonable approximations of the opportunity costs of production factors, and the situations in which it is advisable to correct them and estimate so-called shadow prices. Section 4.7 discusses the interaction of supply and demand to estimate cost changes in the labour market. Section 4.8 covers how to deal with the cost of public funds; and finally section 4.9 analyses the problem of projects that, evaluated with shadow prices, show a positive social NPV but a negative financial result.

4.2 THE FACTOR PRICE AS AN APPROXIMATION OF THE OPPORTUNITY COST

What is the opportunity cost of seeing Eric Clapton? The opening quote of this chapter was the only question included in a survey carried out with students in an academic meeting in Philadelphia. Of the 200 respondents, 45 per cent were from institutions currently ranked in the top 30 US economics departments, one-third of the sample were students and around 60 per cent of the respondents had taught an introductory economics course at university level. Before checking your answer it is worth noting that the result was the following:

A. $0 (25.1%) B. $10 (21.6%) C. $40 (25.6%) D. $50 (27.6%)

The figures show that the respondents appear to be randomly distributed across the possible answers. Only one out of four gave the correct answer. Let us see why $10 is the right one. By going to see Eric Clapton you lose $50, the amount you value going to see Bob Dylan (which is the next best alternative), but save $40, required to go to the Dylan concert. So the actual loss is the net value lost ($50 – $40).

It is worth emphasizing that the value you give to the Clapton concert is irrelevant to answer the question of the opportunity cost of this concert. Moreover we do not know whether you will go to the Clapton concert or not. The only thing we do know is that if you go, the value of the Clapton concert is at least $10.

The cost of a project is what the society loses by giving up a particular set of goods because of its implementation. Such costs are not the goods that we must renounce, but the utility lost by renouncing these goods. This value is, expressed in monetary terms, the amount that individuals are willing to pay for the goods that are no longer produced.
If a bridge is built, its cost is the net value of all the goods we have renounced in return for the new infrastructure. The cost of the bridge would be, in a strict sense, the utility lost by the individuals because of the loss of goods that could have been produced had the factors of production employed in its construction been employed in the best available alternative, instead of the construction of the bridge.

Expression (4.1) reflects this idea:

\[ C_j = \sum_{k=1}^{s} p_k dx_k, \]  

where \( C_j \) is the total cost of producing good \( j \) (i.e. the bridge) and \( dx_k \) is the marginal reduction in the quantity produced of the \( k \) good lost to produce good \( j \) multiplied by the marginal willingness to pay for those goods \( p_k \) (under the assumption of a small project). In practical terms it is very difficult to identify which goods are no longer produced because a particular project is carried out. One solution to this informational problem is to find an approximation in the factor market, where the demand is derived from what is happening in the market for goods, and the factor supply represents the opportunity cost of the factor. In the case of good \( k \) we will assume for simplicity that its production function depends on two production factors \( z_1 \) and \( z_2 \).

\[ x_k = f(z_{k1}, z_{k2}). \]  

The total differential of (4.2) shows that the variation in output depends on the amount of factors used and their marginal productivities.

\[ dx_k = \frac{\partial x_k}{\partial z_{k1}} dz_{k1} + \frac{\partial x_k}{\partial z_{k2}} dz_{k2}. \]  

Substituting (4.3) in (4.1) we obtain the following expression:

\[ C_j = \sum_{k=1}^{s} p_k \left( \frac{\partial x_k}{\partial z_{k1}} dz_{k1} + \frac{\partial x_k}{\partial z_{k2}} dz_{k2} \right). \]  

We know that in a competitive market firms purchase additional units of the production factor while the value to the company of its marginal productivity is greater than its price. In equilibrium the company that produces good \( k \) will employ additional units of factor until:
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\[ p_k \frac{\partial x_k}{\partial z_{k1}} = w_1, \]

\[ p_k \frac{\partial x_k}{\partial z_{k2}} = w_2, \]  

where \( w_1 \) and \( w_2 \) are the factor prices.

Substituting (4.5) in (4.4) and taking into account that the sums of \( dz_{k1} \) and \( dz_{k2} \) are \( dz_j \) and \( dz_{j2} \) respectively, we obtain expression (4.6), which is an operational formula for calculating the cost of the project; however it must be remarked that (4.6) was derived under the assumption of perfect competition in the market for goods. We analyse below the corrections to be made in order to measure the costs of the project when we relax the assumption of perfect competition.

\[ C_j = w_1 dz_{j1} + w_2 dz_{j2}. \]  

Now, with expression (4.6), the cost of the project (the production of good \( j \)) can be calculated from the quantities of factors required for the production of \( j \) (\( dz_{j1} \) and \( dz_{j2} \)) and their respective prices (\( w_1 \) and \( w_2 \)). It should be emphasized that equation (4.6) is valid for small changes in the use of the factors. Later we will also analyse the corrections to be made when the amount of factors required for the project is not marginal, as well as the changes to be made when there are distortions (taxes and subsidies) in the factor markets.

### 4.3 AVOIDABLE COSTS AND SUNK COSTS

Figure 4.1 shows the case of a good supplied by a monopoly with a cost function given by \( C = K + cx \). With this cost function, the average variable cost is equal to \( c \), which also represents the marginal cost. The presence of fixed costs \( K \) (assumed sunk) makes the average cost greater than the marginal cost \( c \). In the case represented in this figure the price charged \( p_0 \) is higher than the average cost \( c^* \). The profits are equivalent to the area \( p_0abc^* \).

Suppose we evaluate a policy consisting of a small change in the price of the product represented in Figure 4.1: what is the change in costs? If there are no capacity constraints, the change in production from \( x_0 \), a consequence of the price reduction, increases the costs \( c\Delta x \) but not \( c^*\Delta x \), because the fixed costs \( K \) do not vary with the policy.

Suppose now that the project consists of closing the firm. What are the cost savings? What is the change in welfare? If one thinks in terms of
surpluses: the consumer surplus is reduced by the difference between what consumers are willing to pay \((\gamma ax_0, 0)\) and what they actually pay \((p_0ax_0, 0)\), thus closing the business reduces the consumer surplus in \(\gamma ap_0\). The change in producer surplus must be added to this quantity.

One could think at first glance that the producer loses the profits he received – the difference between the revenue \(p_0ax_0\) and the production costs \(c*bx_0\) – but this is in fact not so as long as \(K\) is a sunk cost. If \(K\) is sunk the producer loses the revenue \(p_0ax_0\) and saves only the avoidable costs \(c*bx_0\); that is, with the closure he loses the profit \(p_0abc^*\) and the fixed costs \(c*bdc\). The reduction in the social surplus with the closure of the business is equal to the area \(\gamma adc\).

In terms of changes in the willingness to pay and the use of resources, the policy of closing the business does not convert the fixed costs into avoidable ones because they are unrecoverable (sunk) and we can only save the variable costs. The closure of the business involves a loss of gross social benefit equal to the willingness to pay for the quantity \(x_0\) and a saving of resources that is limited to the avoidable costs, resulting in a net loss of social welfare equal to the area \(\gamma adc\).

Suppose now that \(K\) is fixed, but when closing the business the fixed factor can be used for other purposes (i.e. it is not a sunk cost). For example Figure 4.1 may represent a particular airline route with fixed costs corresponding to the aircraft that operate this route. These costs are fixed when \(x > 0\) but they become variable when \(x = 0\) and the route is closed (then the aircraft are assigned to alternative routes). In this case the lost consumer surplus remains \(\gamma ap_0\) but the lost producer surplus is now only
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$p_{abc}$, because the avoidable costs when the business is closed are now $c^*b_{x_0}0$. Now the loss of social surplus is lower: $\gamma_{abc}^*$. In economic evaluation it is very useful to distinguish between fixed and sunk costs because, depending on the project, the variation in the costs may differ substantially, as shown in the previous case. It is not advisable to follow a rigid taxonomy. In the previous example of the airline, the aircraft could be considered as a fixed factor but not sunk. Nevertheless if for example there is an excess capacity of aircraft and when closing the route the aircraft are not needed in any other place, the aircraft costs are sunk.

4.4 INCREMENTAL COST AND AVERAGE COST

The calculation of opening, upgrading or closing a health, recreational or transport service cannot be simply based on the product of the average cost per user by the number of users. A usual source of error in the allocation of costs to services is the use of the average cost rather than the incremental cost. The incremental cost of providing a service is the change in total cost resulting from such an action. A procedure to estimate the incremental cost is to introduce the concept of units of activity; such as number of beds and staff hours in a health service, bus miles and hours worked by drivers in a transport service, and so on.

Consider the following example. A service is provided by the local authority with a total cost denoted by $C$. The expression breaks down the total cost into cost subgroups that may be reasonably related to some activity units:

\[ C = LC + CC + VC, \quad (4.7) \]

where $LC$ is the total cost of the staff, $CC$ is the cost of capital equipment and $VC$ represents the variable costs that depend directly on the number of users (for example a meal served or a dose of medicine). Once the service is analysed it appears that the total costs are a function of the total hours worked ($H$) by the specialists attending the service, the number of units of equipment used ($E$) and the number of users served ($U$), according to the following expression:

\[ C = hH + eE + uU, \quad (4.8) \]

where:

\[ h = \frac{LC}{H}; \quad e = \frac{CC}{E}; \quad u = \frac{VC}{U}. \quad (4.9) \]
If a given project consists of extending a service to a population not covered so far, and the evaluation of this project is based on the average cost instead of the incremental, it is likely that we will incur some mistakes when calculating the cost of this project. Furthermore the error increases with the heterogeneity of the users attended. If the users differ in the ratio of hours of specialist/user and capital/user, by using the average cost we underestimate or overestimate the incremental cost of expanding the service under evaluation.

A procedure to calculate the cost of the project is to identify the type of service required by the targeted population: to determine how many activity units are required and then calculate the cost of expanding the service by using the coefficients $h$, $e$ and $u$. The following numerical example illustrates the consequences of using the average cost rather than the incremental cost in the evaluation of a project, when the users differ in the type of service required.

A public centre that provides health services to a population of 100 users has a total cost of $1000. The breakdown of these costs is as follows: staff costs ($600), costs of equipment ($300) and costs varying with the number of users ($100). The total values of the units of activity on which the costs depend are: 200 hours of specialist input ($H$), 10 hours of equipment ($E$) and 100 users ($U$). There are three different services ($A$, $B$ and $C$) that use the units of activity according to the distribution in Table 4.1.

Given this allocation and taking into account that according to (4.9) the unit costs are $h = 3$, $e = 30$ and $u = 1$, the total and average costs of the three services can be calculated as shown in Table 4.2.

Table 4.1 Cost allocation

<table>
<thead>
<tr>
<th>Services</th>
<th>Units of activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td>A</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>200</td>
</tr>
</tbody>
</table>

Table 4.2 Total and average costs ($)

<table>
<thead>
<tr>
<th>Services</th>
<th>hH</th>
<th>eE</th>
<th>uU</th>
<th>Total cost</th>
<th>Average cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>120</td>
<td>60</td>
<td>70</td>
<td>250</td>
<td>3.6</td>
</tr>
<tr>
<td>B</td>
<td>360</td>
<td>90</td>
<td>20</td>
<td>470</td>
<td>23.5</td>
</tr>
<tr>
<td>C</td>
<td>120</td>
<td>150</td>
<td>10</td>
<td>280</td>
<td>28</td>
</tr>
</tbody>
</table>
With this data, assume that it is necessary to assess a policy consisting of the expansion of a health service to a population of 35 users. This service requires the following units of activity: an additional hour of capital equipment and 20 more hours of specialists. Using the average total cost ($10), the cost of the expansion of the service is equal to $350; however, if one calculates the incremental cost \( \Delta C \), the expansion would cost:

\[
\Delta C = 20h + e + 35u = 125. \tag{4.10}
\]

On many occasions the average cost is a good approximation of the incremental cost but, as shown with this example, the average cost may be misleading and lead to overestimation or underestimation of the actual incremental cost because the project capital/users, hours/users or any other relevant ratios differ from the mean values.

### 4.5 COSTS WITH AND WITHOUT THE PROJECT

Many projects aim to reduce the cost of producing a particular good or providing a service. The cost of water, electricity or transport is reduced by investments that incorporate some new technology or through a capacity expansion. In the quantification of the expected savings it is not enough to compare the costs of the two technologies or the levels of the two capacities. We must take into account the behaviour of demand and determine how the new equilibrium determines the costs that are relevant to the comparison.

With the help of Figures 4.2 to 4.4 it is easy to show how the benefits of a project consisting of the reduction of the user’s cost cannot be predicted by examining the supply side only. Cost savings for existing users do not depend only on investment (new technology, capacity expansion, etc.). The behaviour of the demand is also crucial for the calculus of the social benefit of the project. The interplay between costs and demand and its impact on the \( NPV \) of the project is reflected in the four figures below.

In Figure 4.2 the average cost function \( AC^0 \) shifts to \( AC^1 \) with the project. This cost function shows the time that each consumer invests when using the service or facility in question, for example a road for a given level of demand. For a low traffic volume, travel time does not vary with the number of users. In the equilibrium the generalized price (time multiplied by the value of time, for simplicity) is equal to \( g^0 \). The benefit of a project reducing the cost from \( AC^0 \) to \( AC^1 \) can be decomposed into two parts. Up to \( x^0 \) the benefit is measured as the area between the cost...
function without the project and the new one with the project \((g^0 \text{ad} g^1)\). The benefit of increasing the quantity \((x^1 - x^0)\) is measured as the difference between the willingness to pay \((abx^1 x^0)\) and the incremental cost of meeting that demand \((dbx^1 x^0)\).

The benefits of generated traffic are equal to the area \(abd\) in Figure 4.2 and \(abe\) in Figure 4.3. We are interested here in the differences that occur in the benefits for the existing traffic. In Figure 4.2 there is no congestion without the project and the investment reduces the generalized price of travelling from \(g^0\) to \(g^1\) (e.g. the construction of a tunnel). The potential travel time savings of the project are equal to \(g^0 - g^1\), and given the absence of congestion these potential savings become actual savings. This is not so in the case represented in Figure 4.3, where there is congestion with and
without the project. Now the potential saving of the user \((g^0 - g^1)\) is not the actual saving \((g^0 - g^1)\) since the reduced travel time generates an increase in the numbers of users and then congestion increases resulting in a higher generalized price than \(g^2\).

Now the equilibrium is at point \(b\) with a generalized price \(g^1\) for all users. The difference in costs for the existing quantity is \(g^0 - g^1\), which is lower than \(g^0 - g^2\). If we do not take into account the interaction between costs and demand, we may overestimate the benefit of the project.

An extreme case that illustrates the importance of considering the behaviour of the demand when assessing an expansion capacity project is represented in Figure 4.4. The potential time reduction \(g^0 - g^1\), if the project is undertaken, does not materialize as a result of the increase in demand. Let us consider this case in more detail.

The case represented in Figure 4.4 reflects the idea argued by those opposed to new investments in roads when they reason that the new capacity is absorbed in a short time by new traffic. Suppose there are two roads, \(A\) and \(B\), between two cities. Using road \(A\) (the one represented in Figure 4.4) takes half an hour if there is no congestion (the intersect of \(AC^0\) with the vertical axis), but \(A\) is a narrow road that rapidly becomes congested as shown by the slope of \(AC^0\).

Road \(B\) has a larger capacity and it is never congested, but its layout is not as straightforward as that of \(A\). The trip takes 45 minutes \((g^0)\) regardless of the traffic volume. With respect to other characteristics the two roads are identical and drivers always choose the fastest. Assume that \(B\) has a traffic volume much higher than \(A\); hence, as long as it is faster using \(A\) than \(B\), new drivers will be diverted from \(B\) to \(A\). The perfectly elastic demand function in \(A\) reflects this fact with \(g^0\) equal to 45 minutes.
We can see that, without the project, there is not other equilibrium different from \( a \) with \( x^0 \) users using \( A \) with a generalized price equal to \( g^0 \). When the number of vehicles is less than \( x^0 \), the travel time on \( A \) is less than \( g^0 \) (i.e. less than 45 minutes invested in \( B \)), and thus additional users will leave \( B \) and come to \( A \) until travel time is equalized on both roads. In the same way, traffic on \( A \) cannot exceed \( x^0 \), since in this case the travel time on \( A \) is more than 45 minutes and it would be preferable to switch to \( B \).

Consider a project that increases the number of lanes on \( A \) (represented by a shift in the generalized cost function from \( AC^0 \) to \( AC^1 \)) and remember that traffic on \( B \) is always higher than the volume that \( A \) can absorb. If the traffic is kept fixed at \( x^0 \), travel time saving would be \( g^0 - g_1^0 \), distance \( ad \) in Figure 4.4. We could be tempted to estimate the annual benefits of investment as \( (g^0 - g_1^0)x^0 \), plus the benefits of generated traffic. However the reduction in travel time on \( A \) attracts new users until the travel times on both roads are equalized. The new equilibrium with the project is located at \( b \) with traffic \( x^1 \) and travel time \( g_1^1 = g^0 \), which implies that the project does not generate social benefits.\(^4\)

### Shifts in Demand and Estimation of Benefits

Given the slope of the cost function, the magnitude of the annual benefits after the cost reduction depends, as we have seen, on the reduction in costs with the project and the slope of the demand function. The larger the slope the larger the benefits for the existing demand after the implementation of the project that reduces costs. On the other hand, the smaller the slope the smaller the benefits, with the extreme case of zero benefits when the demand is perfectly elastic (as shown in Figure 4.4).

The interaction between costs and demand does not finish here because demand does not remain constant over time either. Demand shifts because of changes in income or in the population, and each year we must re-estimate the equilibrium that will take place and that will affect the magnitude of the benefits. It is not enough to calculate the benefit the first year and then apply an annual growth rate if the average cost is a function of \( x \) and the demand changes because of exogenous factors.

Figure 4.5 represents an average cost function increasing with the volume of demand. Although the project represents a potential reduction in the generalized price of travel equal to the distance between \( AC^0 \) and \( AC^1 \), the movement in demand after the price reduction generates an increase in the quantity demanded from \( x^0_t \) to \( x^1_t \), where \( t \) denotes the period of time, which results in (more) congestion, making the final
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generalized price \((g^1_t)\) greater than that initially expected. This is what we have seen in Figure 4.3. What happens next year \((t + 1)\)? The main change that we must incorporate is the shift in the demand function. To calculate the benefits in future years it is necessary to determine the equilibrium with and without the project in each of the years during the life of the project.

In Figure 4.5 the demand grows because of exogenous changes in income, population, tastes, and so on in period \(t + 1\), showing again that the change in costs affects the estimation of social benefits. The demanded quantities needed for calculating the annual benefits are now \(x^0_{t + 1}\) and \(x^1_{t + 1}\) for period \(t + 1\), \(x^0_{t + 2}\) and \(x^1_{t + 2}\) for period \(t + 2\), and so on until period \(T\) (the final year of the project’s life).

It can be seen that in period \(t + 1\) the quantity without the project is \(x^0_{t + 1}\), for which the generalized price is higher than that in period \(t\) because of the exogenous change in demand. The relevant level of demand for the measurement is \(D_{t + 1}\), for which the distance between \(AC^1\) and \(AC^0\) is equal to \(g^0_{t + 1} - g^1_{t + 1}\).

4.6 MARKET AND SHADOW PRICE OF FACTORS

In the economic evaluation of projects an input is valued at its market price if this price is a reasonable approximation of the opportunity cost of the resource. Unfortunately the market price of a production factor used in a public project does not always reflect its opportunity cost and some correction has to be made. The name given to the new price is shadow price. Let us see some examples.
Opportunity costs, market and shadow prices

The Cost of Land

A wide range of projects require land and sometimes this land is situated in locations with limited possibilities of substitution. This is the case, for example, with dams and airports. In principle, determining the cost of the land required for an infrastructure investment project should not present major problems as long as the land market operates under competitive conditions and there are no distortions causing the real price to differ from the true opportunity cost of the input.

The opportunity cost of land used in the construction of infrastructures is the net benefit lost in the best possible alternative use of that land. For example, when the best alternative use is in agriculture, the market price of land will reflect the discounted market value (net of variable cost) of agricultural production during the time of use of the land for the project. This price will be higher the more valued the production is in that piece of land and the smaller the possibility of substitution is for other pieces of land. Therefore, if the market is competitive, the price of the land will reliably reflect the opportunity cost.

The determination of the land price in a competitive market with a fixed supply is represented in Figure 4.6. At point \( b \) the supply curve \( S \) and the initial demand for land \( D^0 \) intersect determining the price \( w^0 \), for which all the available land is used for those purposes with a higher willingness to pay. A marginal increase in demand in this market as a result of the project would not significantly change the equilibrium at \( b \), and the price \( w^0 \) could be used as an approximation of the land’s unit cost.

![Figure 4.6  Market of a fixed supply input](image-url)
When the project represents a significant change in the demand for land, such as the case represented in Figure 4.6, the shift of the demand function from $D^0$ to $D^1$ reflects that the project requires the amount of land represented by the distance $ad$, and the equilibrium changes from $b$ to $d$, raising the price to $w^1$. The initial demand for land is reduced to $za$, releasing for the project the extension of land $zb - za$. The opportunity cost of that land is represented by the area $abz_a$, that is, the willingness to pay of the private sector for $zb - za$, a quantity of land that it is no longer in demand because of the price increase.

The area $abz_a$ can be estimated using the initial and final prices and the initial and final quantities:

$$\frac{1}{2}(w^0 - w^1)(zb - za),$$  \hfill (4.11)

or what is the same (the rule of a half):

$$\frac{1}{2}(w^0 + w^1)(zb - za).$$  \hfill (4.12)

We assume here that all other prices remain virtually unchanged.

The application of expression (4.12) is straightforward: we have to multiply the mean of the initial and final prices by the quantity of land required by the project. Nevertheless, determining the opportunity cost of land presents additional difficulties related to the speculation to which the land market is subject and the externalities resulting from the land use in surrounding areas.

Quite often the infrastructure project requires the use of non-marketed land, for example when a highway crosses a natural park with unique wildlife. Economic valuation of the cost of this land requires calculation of the opportunity cost of the input beyond any payment for the expropriations that might be required, estimating the value of what the society is giving up because of the impact of the infrastructure on the original use of the land. Moreover it is not enough to calculate the cost of the land used, because the surrounding areas will suffer the negative effects of this type of project, further increasing the opportunity cost of the land required.

**The Cost of Equipment, Materials and Energy**

The use of capital equipment, energy and materials in virtually all investment projects requires the use of prices that when multiplied by the quantities of the inputs result in the cost of these inputs. The cost–benefit analysis of the construction of high speed lines (see Chapter 10), for example, distinguishes between construction costs, rolling
stock, maintenance and operation. The cost of locomotives, wagons and electric power were estimated from the prediction of demand and the number of trains required to meet that demand. With the number of services needed and the kilometres travelled, energy consumption requirements were calculated.

The prices used to convert these physical units into a monetary value are the market prices net of taxes. This option is applicable to the majority of projects whose demand is marginal in the factor markets or to those for which, without having marginal demand at the national level, there is an international market where at the price \( w_0 \) the supply is perfectly elastic.\(^5\) Figure 4.7 represents the market for an input and it is assumed that the demand of this input for the project is marginal with respect to the total factor market. The shift in the demand function \( D \) is negligible, and the equilibrium price \( w_0 \) is not changed because of the project. The existence of a tax makes the market price \( w_0 \) differ from the opportunity cost of using the input. The shadow price in this case is \( w_0 - \tau \), where \( \tau \) is the tax per unit in absolute value. The tax, in this case, is a mere transfer of income.

4.7 MARKET PRICE AND THE SOCIAL OPPORTUNITY COST OF LABOUR

Virtually any investment project requires the use of labour. The construction of public infrastructure needs workers with different skills. Moreover,
once the construction is finished, additional labour is required for maintenance and operation during the life of the project. Labour is required in the case of a project consisting of restoring the damage to a natural environment, as well as in educational or health-related projects.

Once we have estimated the number of workers required for the project we need a price to calculate the cost of labour. Should we use the gross or the net wage? How should we account for the creation of new jobs in areas of high unemployment?

It is useful to distinguish three possible sources concerning the amount of labour demanded by a project:

- Workers who are already employed in other productive activities.
- Voluntarily unemployed workers at the current wage, who would work for a marginally higher wage.
- Involuntarily unemployed workers, who are willing to work if jobs were available at the current wage.

Once the requirement of labour for the project and the proportions of the three sources have been estimated, we must look for the opportunity cost of labour. Market wages, however, do not always reflect the social opportunity cost of labour. The discussion of the labour market in this section assumes that the project has significant effects changing the equilibrium wage. The aim is to illustrate the ways to obtain the shadow price of labour in different labour market conditions, even if in practice wages do not change significantly with the project. Let us start with the simplest case of a competitive labour market without taxes or unemployment benefit, represented in Figure 4.8.

The factor supply function reflects the marginal value of leisure to the workers and the demand function the value of the marginal productivity of labour ($L$) for the firm. This shows that at the equilibrium wage ($w^0$) the value of marginal productivity is equal to the value of leisure to the marginal worker. If the amount of labour required for the project is marginal we can use the equilibrium wage $w^0$. Suppose on the contrary that, as it is represented in Figure 4.8, the project requires ($L^1 - L^0$) work units, as represented by the movement of the demand function from $D^0$ to $D^1$ at a distance $ad$.

By increasing the demand for labour work from $a$ to $d$ the market wage increases from $w^0$ to $w^1$ and the number of workers employed in the labour market increases from $L^0$ to $L^1$. To see what shadow price should be used in the project it is necessary to know the source of the employees absorbed in the project under evaluation. Figure 4.8 shows how the use of labour increases from $L^0$ to $L^1$, and thus the number of employees $L^1 - L^0$ should be valued by
what the society has lost as a result of employing this labour quantity in the project. Since they were voluntarily unemployed, what is lost when they are employed in the project is the value of their leisure (area $bdL^0L^a$).

What about the rest of the labour employed in the project? Figure 4.8 shows that the $(L^0 - L^a)$ units of work remaining come from other productive activities in which, after the rise in wages from $w^0$ to $w^1$, it is no longer profitable to hire labour beyond $L^a$. The total value of the production lost as a consequence of displacing labour from other productive activities is the social opportunity cost of employing these workers in the project (area $abL^0L^a$).

When there are income taxes, the previous analysis needs to be expanded in order to correct the price of the new workers who are employed thanks to the implementation of the project. Figure 4.9 shows a situation similar to that described in Figure 4.8 with only one additional element, the introduction of an income tax (which for simplicity is presented as a constant tax ($\tau$) per unit of labour), which makes the market supply function ($S$) differ from the marginal valuation of leisure ($S - \tau$). Therefore the social opportunity cost of workers, $L^1 - L^a$, is now $efL^1L^0$ instead of $bdL^1L^0$, since the total amount of taxes $bdef$ is a transfer of income. In the case of workers displaced from other activities, the area $abL^0L^a$ still represents the cost of employing the displaced workers $L^0 - L^a$.

Finally let us consider the case of involuntary unemployment. Figure 4.10 represents a situation in which at wage $w^0$ there are more workers willing to work than those who are currently required ($L^0$). At the level
of demand $D^0$ there is involuntary unemployment. The project represents a shift in the labour demand to $D^1$, which means an increase in the demand for previously unemployed labour, $L^1 - L^0$. What is the social cost of employing the involuntarily unemployed? If there were no taxes or unemployment benefits for these workers, the existing wage $w^0$ would be the social cost of a unit of labour and therefore the total cost would be represented by the area $abL^1L^0$. 

**Figure 4.9**  Shadow price determination in the labour market (with taxes)

**Figure 4.10**  Shadow price with involuntary unemployment
In the presence of income taxes ($\tau$) and unemployment benefits ($\sigma$) the total social cost would be represented by the area $fgL^1L^0$, because tax revenue ($ab$) and unemployment benefits ($def$) are transfers of income and should be deducted. The shadow price of labour is therefore reduced to the marginal value of leisure; this quantity can be obtained by reducing the gross wage of a worker by the amount given by the income tax and the unemployment benefit. That is to say remuneration ($w^0 - \tau - \sigma$) equals the marginal value of leisure. In any case it is what the society loses by employing the worker in the project.

The following numerical example illustrates why a worker would not accept a wage smaller than $w^0$. Assuming that the unemployed worker receives a subsidy of $100 and he values his leisure at $40, the worker will not accept work for less than $150, as after being been contracted he has to pay $10 in taxes. This numerical example can help to distinguish between the private opportunity cost of the worker joining the labour market ($150$) and the social opportunity cost ($40$). The latter is the shadow price of labour to be used in the project.

When there is a minimum wage regulation we also need to correct the market wage to account for the social opportunity cost of labour in the same manner as described above, just looking for what the society loses by employing the workers in the project. This could be approached through the mean of the minimum wage and the minimum reservation wage, as no one accepting work would have a reservation wage lower than the minimum wage, and the reservation wage of anyone employed for the project could be higher than the minimum reservation wage. In the absence of information on the minimum reservation wage, the shadow wage could be estimated as one half of the minimum wage (i.e. assuming the minimum reservation wage is zero).

The previous treatment of the shadow price for labour when there is unemployment does not take into account what happens to the output that is generated by the project. If the production generated by the project is sold in a competitive market and reduces the price, it may happen that labour is reduced as a result of business closures in the private sector of the economy; but it could also be true that the product or service associated with the project is complementary to other competitive markets and therefore it encourages job creation (see Chapter 3). Determining the true shadow price in these cases is not immediate. In any case a careful analysis of the markets closely related to the project and its potential impact on them can help the evaluator (see Johansson, 1991).
4.8 THE SHADOW PRICE OF PUBLIC FUNDS

Many projects require public funds. Sometimes users are not charged for the service supplied, as in the case of a free road, or a revenue generating project that requires some public funding, such as the case of a natural area with an entry fee insufficient to cover the total cost. In both cases they can be partially or totally funded by taxes. The problem of tax revenue from the standpoint of efficiency is that tax collection is not a simple transfer of income between consumers, producers and the government. There is an efficiency loss associated with the operation of transferring funds through taxes that leads to the question of what the deadweight loss of the tax is.

To illustrate why it is necessary to estimate a shadow price of public funds, consider for simplicity the case of an indirect tax (in year 0) in a market unrelated to the project,\(^6\) as represented in Figure 4.11. The tax is intended to finance fully a project whose investment costs \(I_0\) occur only in year 0. It produces constant annual benefits \(B\) during each of the \(T\) years of life of the project. The real interest rate is zero and the benefit \(B\) is entirely consumer surplus (assuming that nothing is charged for the good).

The \(NPV\) of the project seems to be equal to the difference in the flow of benefits and the initial costs:

\[
NPV = -I + TB. \tag{4.13}
\]

Nevertheless a simple look at Figure 4.11 shows that the social cost involved in the finance of the project is greater than \(I\), because it can be seen that the effects of the introduction of the tax are not limited to an

![Figure 4.11 The shadow price of public funds](image-url)
Opportunity costs, market and shadow prices

income transfer. Initially the market is in equilibrium at $b$ with a price $p_0$ and a quantity equal to $x_0$. The introduction of the tax changes the equilibrium of the market by raising the price paid by consumers to $p^+$ and reducing the quantity demanded to $x_1$. The price received by producers is $p^-$ and tax collection is represented by the area $p^+adp^-$, which is equal to $I$.

The introduction of the tax has caused a reduction in the quantity from $x_0$ to $x_1$. This reduction of the quantity represents a negative effect on the economy, as the consumers were willing to pay the area $abx_0x_1$ for goods or services whose opportunity cost is represented by the area $dbx_0x_1$. The net loss of efficiency caused by no longer producing $x_0 - x_1$ equals the area $abd$. This extra cost of obtaining public funds should be included in the evaluation of the project.

Assuming that the area $abd$ is, for example, 25 per cent of the tax revenue, the shadow price (or shadow multiplier) of public funds is equal to 1.25, and the cost of the project is equal to $1.25I_0$. Let $\lambda_g$ be the shadow price of public funds; then we must modify (4.13) in order to reflect the opportunity cost of the investment:

$$NPV = -\lambda_gI + TB,$$  \hspace{1cm} (4.14)

so we can appreciate that, to obtain an $NPV$ greater than zero, the following must be satisfied:

$$\frac{TB}{I} > \lambda_g.$$  \hspace{1cm} (4.15)

The economic interpretation of expression (4.15) indicates that, for a project funded by taxes to be socially profitable, the social benefit obtained per unit of money invested has to be greater than the opportunity cost of the public funds.

The previous argument is based on the assumption that the project is financed entirely by taxes and that it does not have operating revenues and costs. A more general case is represented by the expression:

$$NPV = -I_0\lambda_g + \sum_{t=1}^{T} \frac{CS_t + \lambda_g PS_t}{(1 + i)^t},$$  \hspace{1cm} (4.16)

where:

$CS_t$: consumer surplus in year $t$

$PS_t$: producer surplus in year $t$

$i$: social discount rate.

Expression (4.16) shows that the shadow price of public funds has to be applied to both costs and revenues. The annual net revenue reduces the
need for public funding and therefore the need for taxes, so that $1 collected by charging the users in any year of life of the project has a present value $\lambda_s (1 + \delta)^{-t}$.

### 4.9 SOCIAL BENEFIT AND FINANCIAL EQUILIBRIUM

The use of social benefits in cost–benefit analysis rather than revenues, as in financial analysis, and the use of shadow prices trying to approximate the opportunity cost rather than market prices, raises a fundamental problem: projects whose economic evaluation yields a positive social NPV are often associated with poor financial results; that is, they are not viable from a commercial point of view.

Consider the case of a project consisting of a public service as represented in Figure 4.12. The demand function is $x = 100 - p$. Therefore the demand quantity ($x$) of this service is 100 users per time period when the price ($p$) is zero and zero when the price is $100$.

Suppose the monetary cost of this service is $25,000 (of which $5000 is indirect taxes levied on the building materials used), independent of the number of users for the ten years of life of the facilities. The social discount rate, and the interest rate, are equal to zero.

The social NPV can be expressed as:

$$NPV_s = -20,000 + 10\Bar{B}, \quad (4.17)$$

where $5000$ has been subtracted corresponding to taxes (transfer) to express the cost of the project as an opportunity cost (the actual resources
used in the project are valued at $20,000) and where $\bar{B}$ represents the constant gross benefit in every period.

On the other hand the financial $NPV$ is:

$$NPV_f = -25,000 + 10px,$$  (4.18)

where, in contrast with the social $NPV$, costs are measured at market prices (tax included) and the benefit is reduced to the revenue collected.

Trying with different prices it is possible to calculate the resulting revenue and consumer surplus and consequently the social and financial $NPV$. It can be seen that the highest possible social $NPV$ is equal to $30,000$ and corresponds to a price equal to zero. This maximum social $NPV$ is associated with a negative financial result of $25,000$. The commercial deficit is equal to the fixed costs when the service is free.

With a single price the best financial outcome is achieved when the price is equal to $50$. In this case the revenue is equal to the costs. The improvement in the financial result is parallel to a reduction in the social $NPV$, which is now equal to $17,500$.

This case illustrates the importance of being consistent with the use of shadow prices when evaluating public projects. Suppose we are evaluating the previous project in a context of budgetary restrictions, which require costs to be covered once the service under evaluation has started its operation.

While counting the cost net of taxes in the evaluation we should not forget that the actual price will include these taxes in order to meet the financial constraint. Obviously the estimated demand will not be compatible with a consumer surplus of $50,000$, the one obtained with the free service.

The estimated demand (100) will not be achieved, since the budget constraint requires operation with a higher price than optimal. If the price to be charged to consumers is $50$, the number of consumers is halved, with the corresponding reduction in social benefits. Furthermore, when there are multiple technological options, the choice of the combination of inputs may be biased in favour of inputs with lower shadow prices – which increase the social $NPV$. This choice may, later in the operation of the service, compromise the commercial viability of the project since market prices must be charged.

When calculating the social $NPV$ with shadow prices, the financial implications of these assumptions and the restrictions that have to be faced should be considered simultaneously. The use of shadow prices requires taking into account what will happen later during the life of the project. The use of shadow prices can make some projects socially viable, which would not be feasible using market prices.
An example that illustrates this problem is the construction of a port in an area of structural unemployment. Faced with two alternatives, capital intensive or labour intensive, the use of shadow prices for labour could make more socially profitable the construction of a less capital intensive port. Once the port has been built it is assigned to the port authority with the commercial target of breaking even according to market prices. The paradox arises because the port is placed at a disadvantage with respect to the competing ports that are capital intensive. If the port authority does not receive an equivalent subsidy (which may be prohibited by the rules of competition) that makes the use of labour cheaper (as was shown in the ex ante evaluation with shadow prices), the port will lose traffic in favour of other more efficient ports.

In the economic evaluation of the project shadow prices can be used even if there are no subsidies to reduce market wages. However, if the port will later be exploited under market standards, the demand should be forecast with those prices that correspond to the costs compatible with the inputs valued at market prices.

THINGS TO REMEMBER

- The opportunity cost of an input is the net value lost in its next best alternative. The market price in the factor markets is the first candidate to approximate the opportunity cost. There are circumstances, such as taxes, externalities or unemployment, where the market price has to be corrected to obtain the real opportunity cost. This is what is called a shadow price.
- The incremental cost reflects the opportunity cost of using an input, and the average cost reflects the mean value of the opportunity cost of different units of this input, so the average cost is irrelevant for cost–benefit analysis unless the input productivity is constant.
- The intersection between supply and demand changes over time so the benefits and costs of the first year will usually be different from subsequent years. Examine the equilibrium with and without the project every year during the project’s lifespan.
- Labour is an input, not an output. The opportunity cost of labour in cost–benefit analysis varies, as many other inputs, depending on the preceding use of the input. When a worker is previously employed his gross wage can be used as the opportunity cost. With involuntary unemployment the shadow price can be as low as the value of the worker’s leisure.
• There are many projects requiring public financing. Some projects are provided free and others have charges that are not high enough to break even. When projects have to be financed, partially or totally, by taxes we have to consider the distortionary effect of taxation. Tax collection is not a simple transfer between individuals and the government. There is economic value lost along the way. The deadweight loss of the tax has to be included in the shadow price of public funds.

• The social $NPV$ shows the discounted net social benefit of the project; the financial $NPV$ indicates whether the project generates enough revenues to cover total costs. A private firm is interested in the financial $NPV$, the practitioner of cost–benefit analysis requires the information provided by both of them.

NOTES

2. And sometimes to incur losses of efficiency in some markets to obtain an overall socially desirable result. In certain circumstances it may be optimal to set a price below or above the marginal cost in a regulated market, when there are substitutes or complements for which it is not possible to set a price equal to the marginal cost. This policy is derived from the theory of second best (Lypsey and Lancaster, 1956).
3. In some cases, one must add scrap values. Most buildings and machines have such a value after a particular activity is closed.
4. We might think that investing in capacity in these circumstances is always a bad policy; however there are alternatives to obtaining social benefits with investment projects such as the one described. The introduction of road pricing would produce gross benefits. Moreover, road pricing is an alternative to be considered before investment in new capacity is decided.
5. For the input market with an increasing supply, and where the project causes significant shifts in demand, we can apply the same reasoning as in the labour market (Figures 4.8 and 4.9).
6. For simplicity we assume that the only effects of the tax occur in this primary market. There are no other products or input markets affected by the introduction of the tax. For the effects on other markets related to the primary market by complementary or substitutability relationships see Chapter 3. We also ignore the issue of optimal taxation; that is, the way to collect tax revenue minimizing deadweight losses (see Ramsey, 1927).
5. Economic valuation of non-marketed goods (I)

Only the fool confuses value with price.
(Antonio Machado1)

5.1 INTRODUCTION

The evaluation of positive and negative impacts of projects and public policies is particularly difficult (and controversial) when there is no market for such impacts. Sometimes the project is specifically designed to improve an environmental good or to prevent its deterioration. On other occasions the purpose is of a different nature and the environmental impact is a side effect of the project.

For example, there are safety policies that reduce the risk of death on the job and projects that increase the recreational value of lakes and rivers by improving the water quality. In contrast there are policies such as the promotion of industrial production (or livestock) that contribute to global warming and projects like the construction of an airport, with external effects such as noise or deterioration of the landscape.

Ignoring the negative or positive externalities of a project may hide its true opportunity cost or, equivalently, the true benefits that society obtains from its implementation. If the social profitability of the project is sensitive to the magnitude of the externality it may even change the accept–reject decision, or alter the ranking if the decision is to choose among a set of projects. The problem is how to value the benefits and costs of goods for which no quantities are exchanged in the market and no prices are available for their valuation.

Section 5.2 describes how externalities and public goods are a challenge for economic valuation. The willingness to pay or willingness to accept when there is no market where the good affected by the project is exchanged imposes the need to seek solutions such as the use of ‘ally’ markets or the design of questionnaires that would put individuals in hypothetical situations and allow us to obtain relevant information to estimate the economic value of non-marketed goods. Measurement requires prior understanding
of the key relationships behind the change in the non-marketed good and the well-being of the individuals and these relationships are also biological and ecological.

The monetary measures of changes in individuals’ utility is addressed in section 5.3. A change in the quantity or quality of a public good changes individual well-being, but utility is unobservable and the economist relies on monetary measures of the change in utility. Compensating variation and equivalent variation are discussed, their relationship with the property rights and the reasons explaining why they usually diverge.

There are basically two approaches to the valuation of non-marketed goods: the first based on revealed preferences and the second through interviews with individuals, the so-called stated preferences (Chapter 6). The revealed preferences approach has two main methods, presented in section 5.4, the travel cost and the hedonic price method. In both, the practitioner relies on a related market where information on willingness to pay is obtained.

5.2 THE ECONOMIC VALUATION OF NON-MARKETED GOODS

Externalities and Public Goods

The private cost of a good is the net benefit lost by an economic agent in the next best alternative of the resources used in the production of that good, while in the social cost this net benefit is the one lost by the society. The difference is pertinent when individuals, by consuming or producing goods, affect the welfare of other individuals outside the market transaction. In this case we say that there is an externality, and the private and social costs differ. Externalities can be positive or negative, and can be produced by firms or consumers.

An example of positive externality is associated with a security service hired by an individual who is paying for the service while other residents are benefiting from a safer neighbourhood without paying for it. A negative externality would be caused by a company that pollutes the air and damages the health of residents affected by emissions. In the first case there is an externality generated by an individual that benefits other individuals, while the second is a company that produces an external effect that harms individuals. There may be different possibilities, and they all have in common the difficulty of establishing markets for such goods and, sometimes, the mere absence of them.

The marginal social cost equals the marginal private cost plus the
externality. If there are no externalities or any other distortions such as taxes, the social cost of producing one unit of good equals its private cost. If the externality is negative the social cost equals the private cost plus the extra cost that the externality generates. If the external effect is positive the social cost will be lower than the private cost. The difference between social and private marginal cost (i.e. the value of the externality) does not need to be constant and may increase or decrease with the scale of the activity.

When an individual consumes a private good another individual cannot enjoy that good (rivalry) and the supplier can exclude the individual if he refuses to pay the price (excludability). We have argued that the production or consumption of some goods affects other individuals outside the market, but there is an extreme case of non-rivalry and non-excludability that is called a pure public good. If the good is non-rival and non-excludable, it cannot be produced and sold in the market. Lighthouses have been the classical textbook example.

In cost–benefit analysis there are many cases of changes in the provision of a public good (such as the reduction of air pollution or the enhancement of the quality of the water of a river) that need to be evaluated, though there are no market transactions where we can obtain information on willingness to pay.

Sometimes the change in the level of the public good is not the primary target but the project produces side effects that change that level. This is the case with public works that reduce the visibility of scenic views. Hence the social cost, represented by the reduction in the pleasure of contemplating the landscape, is a cost of the project. Another completely different issue is the difficulty of sensibly expressing this cost as a number for the calculation of the net social benefit of the project.

Other important non-marketed goods in cost–benefit analysis are related to health and safety. Investment in road safety has as a primary objective: the reduction of the number and severity of accidents. The output of this type of project is measured in the number of lives saved and injuries avoided. Other projects affect the health status of people though this is not the primary objective of the project, for example a power station that increases the production of electricity and reduces the quality of air. In both cases the welfare of the affected individuals changes, and the economist trying to calculate the economic value of these effects has to overcome the problem of the absence of markets for these goods.

In our simplified society a change in the natural resources and environmental goods, as well as in the level of risk, affects consumers and producers through the market. This is the case with an increase in the quality of water in a reservoir that reduces the prices of agricultural products. In other cases the consumer benefits directly, and not through a market transaction; for
instance when the government provides more recreational areas in a city. On other occasions individuals are affected through a market transaction in which they do not participate as sellers or buyers, as happens in the case of an airport that produces noise suffered by people who are not direct users. We have called this group the ‘rest of society’ to distinguish the external effects not accounted for in consumer and producer surplus (see Chapter 2).

Moreover, when considering the owners of the production factors, the provision of a public good can increase the rent of a factor of fixed supply, as happens in the case of the restoration of a polluted natural area that increases the value of the surrounding private land within the boundaries. Finally the taxpayers will in most of the cases be negatively affected, bearing the investment and maintenance costs.

It is worth emphasizing here that individuals can belong to more than one group in our simplified society and hence their welfare can be affected through different channels because of their several ‘identities’. The introduction of a clean air regulation can make the individual better off as a citizen who belongs to the group ‘rest of society’ or as the owner of the land, and worse off as a producer and as a consumer of electricity or a tenant now paying a higher rent for his house.  

The practical side of this argument is that the analyst needs a model before going into the quantification of benefits and costs. Adding the change in consumer surplus, producer surplus and ‘rest of society’ can lead to double or triple counting if the change in consumer surplus has been obtained by asking individuals how much they are willing to pay for the implementation of the clean air regulation. In this case the reduction in profits of the firms affected by the stricter policy or the higher quality of the air they breathe will normally be included in their answers. The individual is not only a consumer. He can also be a shareholder and so the question this individual is asked must be clearly formulated before we make use of any resulting number. This gives us a clue as to the difficulties associated with the economic valuation of non-marketed goods based on the responses obtained by surveying individuals about their monetary valuation of changes in the natural resources, the environmental attributes or the level of risk in their lives.

**The Effect on Welfare of a Change in the Non-Marketed Good**

The understanding of the relationship between the change in the environmental good and social welfare is not restricted to its economic content. Some knowledge is required of the biological and ecological links behind the changes in the quantity and quality of the environmental and resource service flows. Freeman (2003) proposes to view the economic value of these flows as the product of three sets of functional relationships.
The first is the link between the human intervention and the change in the environmental good (e.g. biomass of some species for commercial or recreational use or quality of the air). Two types of human interventions are relevant here. One occurs in the market when a factory pollutes the air. The other is the government actions to regulate the commercial activities that affect the environment (e.g. internalization of externalities through Pigovian taxes) or actions that directly focus on the protection or enhancement of the environmental good (e.g. the creation of wildlife sanctuaries).

The effect of human impact on the environmental good has two channels, one is direct, such as the increase in fauna when land use is restricted through a government regulation, and the other is indirect, such as the behaviour of private agents changing with the new regulation. The reaction of the individual and firms has to be contemplated before predicting the final effect of the government action on the environmental good.

Once the change in the quality or quantity of the environmental good is established a second relationship is required in order to know the effect on the human uses of the good. The uses of the resources can happen in the market, as is the case with commercial fishing, or could be the level of quality of the resource for recreational activities (e.g. number of days of acceptable quality of the water for sport activities in a river). Again, there is a direct effect on the output (e.g. tons of fish caught and days of recreational activity) and the level (quantity or quality) of the environmental good, and an indirect effect as the output is also affected by the use of other inputs beside the environmental good, and the quantities of those other inputs vary as a response to the change in the level of the environmental good (e.g. climate change affects the crop indirectly as farmers change the use of inputs as a reaction to the new conditions of production).

The third relationship is the link between the output, or resource flow, and welfare. This consists of the economic valuation of the change in the environmental good. The aim of the economist is not to measure the change in the number of days the river is open for recreational purposes or some physical indicator of the pollutants in the air after government action to improve the environment. The objective is to measure the change in social welfare, and this may include two types of value, the use value and the non-use value of the change in the environmental good.

5.3 WILLINGNESS TO PAY AND WILLINGNESS TO ACCEPT

The problem with goods such as clean air, silence and scenic views is that there is no market in which they are traded and therefore a variation in the
level of atmospheric pollution, noise or visual intrusion associated with a project is difficult to measure in economic terms.

In the absence of a market in which to obtain the valuation we must look for other related goods for which a market exists that reflects the economic effects of the changes in the good not directly subject to market transactions. For example the real estate market reflects in housing prices the noise that is involved in the operation of a nearby airport or the provision of parkland. When there is no possibility of using an ‘ally’ market the alternative is to ask individuals directly for the economic value of the effect on their welfare. The first case is called revealed preferences and the second stated preferences.

Starting from the indirect utility function $U = V(P, M)$, where the utility ($U$) of the individual depends indirectly on the prices ($P$) of goods consumed and his income ($M$) (see Chapter 11), we incorporate a public good (Johansson, 1993) so that, according to (5.1), we can express the individual’s utility as a function of the price vector of private goods, the level of provision, defined in terms of the quantity or quality, of public good ($g$) and his income:

$$V(P, g, M).$$  \hspace{1cm} (5.1)

Suppose that a given project improves the public good $g$; we assume it is environmental quality. Then, suppose the project increases air quality or reduces noise. The initial utility of the individual is represented as $U^0$ and the final utility, once the project has been carried out, as $U^1$ (the superscripts 0 and 1 indicate without and with the project respectively):

$$U^0 = V(P^0, g^0, M),$$  \hspace{1cm} (5.2)

$$U^1 = V(P^0, g^1, M).$$  \hspace{1cm} (5.3)

The increase in individuals’ welfare that occurs as a result of the enhancement in the public good is equal to $U^1 - U^0$. Ideally the change in utility resulting from the project is what we should measure, but the utility is not observable and economists, unable to measure utility directly, rely on monetary measures of changes in utility (see Chapter 11). There are two approximations for obtaining individual valuations of changes in the non-marketed good: the willingness to pay and the willingness to accept.

One way to measure in monetary terms the utility change experienced by an individual is to ask how much money he would be willing to pay for the improvement. This is the compensating variation ($CV$) and, assuming that the individual responds with sincerity, he would be willing to pay an
amount \((CV)\) that would leave him indifferent with respect to the initial situation without the improvement. Expression (5.4) captures the idea of how to measure the improvement in the public good:

\[ U^0 = V(P^0, g^0, M) = V(P^0, g^1, M - CV), \tag{5.4} \]

where it is assumed that the individual’s income does not vary with the project.

Another way to measure the increase in an individual’s utility in monetary terms is to ask the individual for the minimum amount of income that he would accept to forgo the improvement. This is the equivalent variation \((EV)\) and, assuming that the individual responds with sincerity, he would be willing to accept an amount that would leave him indifferent with respect to the situation he would have reached with the improvement. Expression (5.5) incorporates the application of the concept of \(EV\) to an improvement in the public good:

\[ U^1 = V(P^0, g^1, M) = V(P^0, g^0, M + EV). \tag{5.5} \]

An example will help to clarify the difference between the two concepts and their practical use. Consider the case of an individual whose hobby is fishing in a river that will be contaminated if the project of constructing a factory is approved. The expected fall in the quality of life of this individual has to be valued for its inclusion in the evaluation of the project that involves the river pollution among its social costs; the absence of a market for the good ‘clean water’ creates the difficulty of valuation of this damage. If you ask the fisherman how much he is willing to pay to avoid the damage, he would probably angrily answer zero and that, in any case, we should compensate him for the damage.

Consider now the case in which the factory has the right to pollute. The question of how much the individual would be willing pay to avoid the damage now has another connotation to the respondent and, if he answers truthfully, the response would be, within the limits of his income, the maximum that would make him indifferent to continuing to fish in unpolluted waters.

Assume alternatively that the individual has the right to uncontaminated water. The question that would now make sense is: how much is the minimum compensation he would accept for the pollution of the river? Depending on the importance that the individual gives to fishing, and the closeness of substitutes, he would require a larger or smaller amount of compensation; compensation that, in this case, would not be limited by his income, so it could be infinite.\(^4\)
The case of the fisherman shows that the measurement of external effects is not unique and it depends on who has the property rights. Suppose alternatively that the individual fishes in a polluted river and that the project substantially improves the fishing conditions. If the property rights of clean water belong to the fisherman, the valuation question is how much he would be willing to accept to continue with the river polluted (i.e. not to undertake the renovation project). Here, there would be no income limit to the compensation. In the case in which the factory has the right to pollute, the question is how much the fisherman is willing to pay for clean water. Now the response is limited by the individual’s income (Table 5.1 represents the various possibilities).

This example shows that in some questions there is a limit of income and in others there is not. Furthermore there is no reason to expect that the amounts of income accepted or paid by individuals in their responses are equal, as the standard of living of the individual varies with the type of question to answer, and if the marginal utility of income is not constant, the compensating and equivalent variations do not coincide (see Chapter 11).

According to the modelling of the society made in Chapter 2, a project that enhances environmental quality affects different social groups, for example the group ‘rest of society’ (better environmental quality enjoyed by excursionists), ‘producers’ (higher profits because of cost reductions derived from the highest quality), ‘consumers’ (through the lower prices of goods supplied by producers with lower costs) and ‘taxpayers’ (who finance the project through taxes).

In practice the correct formulation of the question that induces the individual to calculate the monetary valuation of the change experienced by an environmental good is critical. If the individual is asked about willingness to pay for the environmental improvement, without any qualification, we will probably obtain a value that includes the individual’s surplus as a consumer but also as a producer or as a taxpayer. Unless the individual is informed that his valuation of the change in the environmental good

<table>
<thead>
<tr>
<th>Who has the property right to clean water?</th>
<th>Fisherman</th>
<th>Factory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean river to be polluted</td>
<td>CV/WTA</td>
<td>EV/WTP</td>
</tr>
<tr>
<td>Polluted river to be cleaned</td>
<td>EV/WTA</td>
<td>CV/WTP</td>
</tr>
</tbody>
</table>

Note:  
CV: compensating variation; WTA: willingness to accept; EV: equivalent variation; WTP: willingness to pay.
should exclude the change in his income as a shareholder or taxpayer, the sum of the surpluses of the social groups that make up our simplified society in this chapter would be incorrect.  

We have seen that $CV$ and $EV$ may be different (see Chapter 11 for a more detailed explanation), but the observed disparity between willingness to pay ($WTP$) and willingness to accept ($WTA$) is larger than the income effect would explain. An empirical study (Horowitz and McConnell, 2002) concluded that there are significant disparities: the differences are minimal for private goods, the ratio $WTA/WTP$ is about 10 for public goods and the explanation cannot be found in survey related problems. 

Other ideas have been presented to explain the disparity; for example substitutions effects in the case of goods with hardly any substitutes available, as happens with some environmental goods. If the possibilities of substitution are scarce higher compensation will be required for a reduction in the quantity or quality of the environmental good (Hanemann, 1991). Other explanations refer to concepts like loss aversion and reference dependency meaning that the individual gives more weight to a loss than to an identical gain (see Kahnemann and Tversky, 2000). Another argument is based on the idea of irreversibility in the context of uncertainty (see section 8.2). The additional cost of killing an option (when the environmental good is not kept at the present level) could be reflected in the higher compensation required for the reduction of that level (Zhao and Kling, 2001).

5.4 VALUATION THROUGH REVEALED PREFERENCES

There are two approaches to measuring the benefit of a project involving an environmental good: the measurement based on revealed preferences and the one based on stated preferences; that is, surveying individuals to seek information about their preferences.

The simplest situation occurs when the change in the non-marketed good is an input for the production of a marketed good. For example, when a public project improves the quality of water used as the input for a residential water supply firm, the benefits of the provision of the public good (water quality in reservoirs or underground) affect the cost function of the water supply firm because they reduce the average cost of producing drinking water. This cost reduction interacts with the demand function, affecting the utility of consumers if prices fall (assuming that water quality, as an output, remains constant) and the utility of the owners of production factors if their surpluses increase.

The problem of not having a market for the valuation of a public good
can be illustrated with two examples of actions that affect environmental goods. The first consists of a municipal project to provide the city with a green area. The second is the construction of an airport that increases the noise in the surrounding residential area.

In both cases the evaluator needs to know the change in the social benefit with the project: in one case the benefit of the individuals derived from the new green area; and in the other case the loss of social benefit as a result of the noise produced by the aircraft. In the second case one might think that, as we know how much it costs to soundproof a house, we could estimate the cost of noise by calculating the cost per household and multiplying by the number of houses within the noise map. Is this really the cost of noise? Actually not, because exposure to noise cannot be avoided when the individuals go outside or open their windows.

The problem with defensive expenditures, as well as averting behaviour (i.e. costs individuals incur in actions to avoid noise or a higher risk of illness) is that they do not measure the value of the change in the non-marketed bad (e.g. noise) but a lower bound of the maximum willingness to pay of the individuals to avoid the loss of utility.

In both the cases of the green area and the noise, we can try to estimate the economic value of the non-marketed good by observing behaviour in an ‘ally’ market, for example the property market, or in the case of health risks the labour market. This method aims to obtain information on the use value of the environmental good and therefore cannot provide information on the non-use value.

The Hedonic Prices Method

An approach based on revealed preferences is to see how noise affects the price of housing, comparing prices of houses near to the airport with others of similar characteristics but different levels of noise exposure. The starting assumption is that among a set of similar houses, individuals pay more for those that are less exposed to noise, we can obtain a monetary measurement of willingness to pay to avoid it, and infer from that price difference the implicit price of noise for individuals.

This approximation is based on the conception of the good as a set of attributes (Lancaster, 1966). Thus a house is described as a composite good of a number of rooms, size of garden, views, quality of construction, location, and so on. If we also add to this that the houses have different levels of noise exposure, we can find the implicit price of noise.

The problem is that the price difference between a house exposed to noise and another house that is not exposed cannot be explained solely by this fact, because the airport also benefits the area by increasing, for example,
the accessibility to the city centre with access roads or by building new subway lines that are part of the airport construction project. Moreover those who have chosen to live near the airport may not have the same preferences and therefore the disutility of noise is not representative of the average individual. Let us see how this method tries to solve these difficulties.\(^6\)

Suppose the price of housing \((P)\) is a function of the following variables:

\[
P = f(R, N, A, F, O),
\]

(5.6)

where:

- \(P\): housing price
- \(R\): number of rooms
- \(N\): noise level
- \(A\): access time to city centre
- \(F\): characteristics of the area (crime rate, number and quality of schools, etc.)
- \(O\): other factors.

Once we have a representative sample of houses, we estimate the function (5.6) whose functional form is often assumed to be logarithmic:

\[
\ln P = b_0 + b_1 \ln R + b_2 \ln N + b_3 \ln A + b_4 \ln F + b_5 \ln O.
\]

(5.7)

First we want to know how the price of housing varies with respect to changes in the noise level, and equation (5.7) allows us to estimate this relationship. If we compute the derivative of the logarithm of price with respect to noise level, holding the other explanatory variables constant, we obtain the elasticity of price with respect to noise level:

\[
\frac{\partial \ln P}{\partial \ln N} = b_2,
\]

(5.8)

or

\[
b_2 = \frac{\partial P}{\partial N} \frac{N}{P}.
\]

(5.9)

In expression (5.9), \(\partial P/\partial N\) is the change in the price of houses when the noise level changes, which is called the implicit price or the hedonic price of noise. Solving for \(\partial P/\partial N\) in (5.9) we can obtain the value of the hedonic price of noise:

\[
\frac{\partial P}{\partial N} = b_2 \frac{P}{N}.
\]

(5.10)
As shown by expression (5.10) the price of noise depends on the parameter estimate (which is expected to be negative), the price of housing and the level of noise. Therefore the hedonic price of noise is not constant unless $P$ and $N$ always vary in the same proportion, which is not what is known from empirical work, as the price of the private good does not have a linear relationship with the environmental quality at the observed levels.

It is worth recalling what measures $\partial P/\partial N$. This partial derivative is the price of noise understood as how much the price of the house decreases when the noise increases in the area where it is located. This hedonic price is therefore the net present value of the negative externality during the lifespan of the house. The externality is therefore measured by a single figure for the whole period. A note of caution on the risk of double counting is necessary since any other measurement of noise damage in individuals would already be accounted for in the loss of home values, assuming, of course, perfectly informed individuals.

The problems with this method are associated with the difficulties of information on long term damage to health, selection bias, omitted variables, sufficient variability of the observations, and so on.

**The Travel Cost Method**

The travel cost method is also based on the use of ‘ally’ markets: the market of goods related to the non-marketed good under evaluation. The idea is to estimate a demand function that captures what the visitors who come, for example, to a wildlife park are willing to pay for its use. Total willingness to pay, which includes the admission fee, travel costs (gasoline, for example), spending on equipment (fishing and climbing equipment, etc.) and time spent on the trip from their place of residence. The population is constituted by the individuals who visit the wildlife park.

It is often used to measure the benefits that individuals gain from the enjoyment of recreational activities and outdoor sports such as fishing, rowing or simply visiting the parks. The farther away the visitor lives the more expensive the visit. There will be a distance for which the *generalized price* is so high that the demand is zero. Individuals make a different number of visits according to the distance, the closeness of substitutes, income, and so on. The researcher’s task is to obtain a demand function that relates the number of visits and the generalized prices they pay by having to travel from longer distances.

Consider the case of a natural park that receives individuals from different concentric zones increasingly distant from the park. Suppose we have identified those areas with their respective population, income,
education, age, and so on. The number of visits to the park from zone $i$ can be expressed as:

$$Q_i = f(C_i, H_i, F_i, O_i),$$  \hspace{1cm} (5.11)

where:

$Q_i$: number of visits originated in zone $i$
$C_i$: generalized cost of travel from zone $i$
$H_i$: population of zone $i$
$F_i$: characteristics of the population of zone $i$
$O_i$: other characteristics of zone $i$.

In expression (5.11) the generalized travel cost $C_i$ can be expressed as:

$$C_i = cD_i + v_iT_i + P,$$

where:

$C_i$: generalized cost of travel from zone $i$
$c$: cost per kilometre
$D_i$: distance from zone $i$
$v_i$: value of time in zone $i$
$T_i$: travel time from zone $i$
$P$: park entry fee.

Once we have estimated an equation relating the number of visits to the explanatory variables according to (5.11) the results usually show that the number of visitors is lower when the generalized cost increases. The next step is to assume that by raising the admission fee the number of visits will be lower according to the estimated coefficients from the sample that includes all the areas that generate trips to the park. In this way we construct a market demand function with the number of visits on the horizontal axis and the price of entry on the vertical axis, whose points are obtained by raising the price and seeing how the increase in cost affects the number of visits. The maximum reservation price is the one that would make the number of visits equal to zero.

It should be emphasized that this hypothetical demand function is constructed under the assumption that the observed relationship between the number of visits and the change in generalized cost (with a constant admission fee) can be extended to the relationship between the number of visits and the entry fee. Once we have this demand function the estimation of the surplus that individuals obtain from their visits to the park is immediate.

The travel cost method is therefore based on the valuation of a good for which there is no market (visits to the park) through the cost of accessing
it (entry ticket, travel costs, equipment, etc.). Just as with hedonic prices, the travel cost method assumes weak complementarity between the environmental good and private goods that are jointly consumed, which means that, when spending in private goods is zero, the utility derived from the environmental good is zero; so the travel cost method measures only the value of use, ignoring passive use value. The assumption implies that the value of a natural park is only related to its use. With this method, the value of the park is zero for non-visitors.

**THINGS TO REMEMBER**

- The reduction of accidents or the enhancement of an environmental good can be the main objectives of a project. In other cases, changes in the number of accidents and in the quality of some environmental goods are side effects of projects that pursue the fulfilment of completely different objectives. In both cases we need to measure the benefits and costs of non-marketed goods.

- It is important to have some knowledge of the physical and biological relationships between the environmental impact and the final effect on individual well-being; otherwise some key elements might be overlooked, affecting the validity of the results.

- The economic valuation when there is no market where the good is exchanged follows two basic approaches. One is based on the revealed preferences, using ‘ally’ markets; the other consists in the design of surveys that would lead individuals to face hypothetical trade-offs to obtain their willingness to pay or accept with respect to changes in a particular non-marketed good.

- The two main methods, based on revealed preferences, for the economic valuation of the change in the resource or the environmental good are the hedonic price and the travel cost. The estimation of the economic value through the costs associated with defensive expenditure and averting behaviour can only be considered a lower bound of the maximum willingness to pay of the individual to avoid the negative effect of the impact.

- In the travel cost and hedonic price methods the analyst works with an ‘ally’ market where some useful information is revealed about the willingness to pay of the individuals. This approach for the economic valuation of the non-marketed good reveals information on the use value of the environmental good; hence, if the good had a non-use or passive value, the total economic value would be underestimated.
NOTES

2. The lighthouse example was questioned by Ronald Coase after reviewing the history of lighthouses and finding that they had been privately operated in England and Wales, though others argue that this only means that there are alternative mechanisms for the provision of public goods (in this case, the government allowed charges levied on ships that used nearby ports). Global Positioning System (GPS) may be a more updated example of public good. There is probably, an almost infinite number of variations, ranging from a pure public good to a strictly private one. A bridge is an example of a variation where drivers may be excluded unless they pay a fee. However, as long as the marginal cost is equal to zero, the use of the bridge is non-rival and the outcome is inefficient. Then, when demand is high enough, congestion appears and the marginal cost is positive.
3. In the extreme case of a world of identical preferences only the owner of the fixed factor would benefit from this regulation. See the excellent chapter 'The indifference principle' in Landsburg (1993).
4. An example illustrating why infinite compensation might be needed is where there is a high risk of dying. Typically you are not prepared to take large risks even if the compensation goes to infinity.
5. For a rigorous analysis of how to express the equivalent and compensating variation when it results in questions posed to individuals, see Johansson (1993).
6. Two common assumptions in the hedonic price method are that the individual’s utility function is weakly separable and that weak complementarity exists. The first assumption implies that the marginal rate of substitution between housing and noise level does not depend on the quantities of other goods. The second assumption is that if housing demand is zero, so is the willingness to pay to avoid noise. The value of passive use or non-use (section 6.2) of environmental quality is not captured by this method because it is not reflected in the price of housing.
Disaster demands a response, but it is often the wrong one. That is what the experience of Sir Bernard Crossland, a safety expert who led the inquiry into a disastrous underground railway fire in London in 1987 which killed 31 people, suggests. This week Sir Bernard questioned the £300m ($450m) spent on fire-proof doors, metal escalators and suchlike on London’s underground after the disaster. The money, he said, might better have been spent on putting smoke detectors in people’s houses. It would have paid for one in every house in the country. House fires kill around 500 people a year, mostly in homes without smoke detectors.

(From The Economist, 11 September 2003)

6.1 INTRODUCTION

When economic measures of changes in the non-marketed good cannot be derived from direct observation of individuals’ choices, economists try to obtain this information by asking the individuals directly through carefully designed surveys to reveal the monetary value of changes in the environmental good or in the safety level. This is specially the case in the so-called non-use value or passive value (willingness to pay for the existence or the conservation of an environmental good with independence of its direct use).

There are two opposite positions with respect to measurement of the economic value of the environmental. One rejects the possible quantification in monetary terms of changes in the environmental good or in the probability of physical risk (having an accident); the other admits the possibility of assigning monetary values to air pollution, the preservation of wildlife or a reduction in the risk of dying.

Section 6.2 explains the concept of non-use value and the attempt to measure it through the contingent valuation method, a survey based technique that creates a hypothetical market where the practitioner tries to elicit values through a questionnaire. We present the main elements of this stated preference method, carefully designed to avoid the most obvious biases, as well as the position of the economists who consider these design efforts are not sufficient to modify the hypothetical nature of the exercise.
The stated preference approach is not reduced to contingent valuation. When the non-marketed good is defined as a set of attributes, conjoint analysis is an alternative to elicit individuals’ preferences. Section 6.3 describes one of the most used methodologies for the application of conjoint analysis, the so-called choice experiments consisting of the description of the good according to the selected attributes and levels. The respondent has to choose between options which can include a price. From the trade-offs revealed by the interviewee, it is possible to estimate the monetary valuation of different attributes.

In section 6.4 we discuss some issues concerning individual willingness to pay and social welfare. First, the treatment of altruism in cost–benefit analysis mainly concerned with the possibility of double counting if the altruistic motive is added to the purely selfish valuation. Second, the ethical problem associated with the monetary measures of some unique environmental goods, impacts on future generation or human life. Third, when individual preferences may be distorted and a conflict arises between the satisfaction of individual preferences and individual welfare.

The economic valuation of changes in physical risk is presented in section 6.5. The value of saving a statistical life is addressed as well as the methods most commonly used for its estimation. Changes in the probability of accidents can be converted into numbers of injuries and deaths and by observing people’s trade-offs between safety and other goods, or through stated preferences it is possible to elicit individual values with respect to changes in the level of risk. Finally, in section 6.6, we discuss the use of values obtained in other studies to avoid the costs of conducting specific surveys and its potential drawbacks.

6.2 VALUATION THROUGH STATED PREFERENCES: THE CONTINGENT VALUATION METHOD

Passive Use Value

The economic value of an environmental good cannot be reduced to what its direct users are willing to pay. The value of this good is higher than its valuation for the direct users (e.g. those who walk in a natural area). For environmental economists the value of non-use (or passive use) is the value that individuals attach to environmental goods that they do not use directly, though they would be willing to pay a positive amount of money for their conservation.

The total value consists not only of the use value, bequest and option
values (to have the possibility of future use). The endangered species or national parks also have intrinsic or existence value, not necessarily related to their direct or indirect use by individuals (Pearce and Turner, 1990). The passive use value of environmental goods and the various positions that can be taken against the possible monetary valuation can be illustrated by the example of a public policy consisting of declaring an area as protected for endangered fauna or leasing the area for mining. If one opts for the natural reserve the public will not be allowed to visit it, so the social benefit is only derived from the passive use value (Carson et al., 2001).

One position taken with regard to the possible quantification of passive use value is that passive use value cannot be expressed in money. Although non-use value has to be taken into account in, for example, the decision to declare the area a nature reserve or for mining, the decision maker assisted by experts will make the choice. Alternatively, the cost–benefit analysis of the project must include the benefits that individuals gain from the passive use of the resource, and these benefits can be monetized by asking individuals their willingness to pay for the existence of the nature reserve.

The report from NOAA on the contingent valuation method, which is often cited as the original academic support for the valuation of the non-use value of environmental goods, justifies the concept of passive use of natural resources and their monetary valuation. The aim of the report was to evaluate procedures for the assessment of environmental damages resulting from oil spills into the sea.

The authors distinguish between use value and passive use value. The first can be identified and measured through the information contained in market transactions. Damage to the professional fishermen and loss of revenue from tourism are easier to estimate than damages to fishermen and other sports people who visit the place in which oil spills occur, although such damage can also be estimated with somewhat more difficulty.

The losses of direct users are what is known as the use values because they are enjoyed by those who make active use of the damaged resource. Nevertheless, as stated in NOAA (1993, p. 4602), these are not the only losses associated with the environmental impact, as:

...for at least the last twenty-five years, economists have recognized the possibility that individuals who make no active use of a particular beach, river, bay, or other such natural resource might, nevertheless, derive satisfaction from its mere existence, even if they never intend to make active use of it.

This concept has been denominated as the existence value and it is the main component of what is now called non-use value or passive use value.
The limitations of methods based on revealed preferences to determine the value of a non-marketed good, especially with regard to passive use value, led economists to use surveys in which, by creating a hypothetical market, individuals are asked directly how much they are willing to pay to preserve the conservation of endangered species or to prevent a negative impact on a scenic view (or to avoid an increase in health risk).

Economists have made great efforts to assess in monetary terms the damage to environmental goods, and the benefits of specific policies for the conservation and improvement of such goods. The sceptics of the methods based on surveys for the valuation of environmental goods point to the origin of the data (stated preferences), as many economists prefer data obtained by observing market behaviour. Nevertheless, this bias in favour of the estimates obtained by direct observation of behaviour is not shared by other social sciences (Carson et al., 2001).

It does not seem, however, that the resistance to accept this valuation method and its results to decide on the approval of projects is limited to the use of stated preferences in a simulated context. The theoretical and practical difficulties are significant, and the positions that some prominent academic economists take on this method of valuation cannot be ignored.

The need to value the environment beyond the use value of the resource seems obvious when it comes, for example, to the valuation of damage from oil spills into the sea. It does not seem reasonable to limit the economic value of the damage to the compensation that insurance companies must pay to those directly affected. The environmental good is a public good, it is neither excludable nor rival. Many individuals would be willing to pay for the conservation of the oceans free of contamination despite not enjoying direct use.

**Contingent Valuation**

The contingent valuation method tries to capture the non-use value of environmental goods; value that, as we have seen, is not captured by the hedonic price method or the travel cost method. In section 5.3 we reviewed the concepts of compensating and equivalent variation and their relationship with the willingness to pay and to accept, and how the use of one or the other depends on the status quo and the allocation of property rights. In the practice of contingent valuation the willingness to pay is generally used in order to make the exercise of valuation more credible. Basically this method relies on asking individuals how much they are willing to pay for an action that prevents particular environmental damage or improves the existing situation.

Hanemann (1994) suggests that in research the details matter and
therefore the way the survey is designed and conducted is crucial, and there
is now accumulated experience and good practice to carry it out sensibly.
The point is how to capture the value that the environmental resource has
to the individual. Two key developments have been that the individual is
faced with a specific and realistic situation rather than abstractions, and
that he tries to respond to closed-ended questions that simulate the context
of voting in a referendum.

Since the objective is to measure the preferences of individuals, the
good practice of contingent valuation avoids general questions like: How
much would you be willing to pay to conserve the forests? According to
Hanemann (1994, p. 22) such questions are meaningless: ‘what is meaning-
ful is paying higher taxes or prices to finance particular actions by some-
body to protect a particular wilderness in some particular manner’.

In this quest to simulate actual situations and to escape abstraction,
Hanemann recommends avoiding a counterfactual, such as ‘What would
you pay not to have had the Exxon Valdez oil spill?’ Because the situation
is irreversible, it is an extremely hypothetical situation. Faced with this
question, the following alternative is much more tangible: ‘What would
you pay for this new programme that will limit damage from any future
oil spill in Prince William Sound (the place where the Exxon Valdez oil
spill occurred)?’

The compensating variation (CV) or maximum willingness to pay for
the project (environmental improvement in the discussion below) could be
obtained from the open question that reflects expression (5.4), where \( g \)
is an environmental good that will experience an improvement through the
project. Gathering this information for all individuals we would have the
monetary expression of the change in utility caused by the project.

The problem is that this open question did not lead to good results in the
past and it was replaced by the closed question in expressions (6.1), (6.2)
and (6.3), where \( \Omega \) is a quantity asked by the interviewer that varies in dif-
ferent subsamples. If \( \Omega > CV \) the individual prefers the situation without
the project; if \( \Omega < CV \) the individual prefers the project; and if what he is
asked to pay is equal to its reservation price he will be indifferent.²

\[
U^0 = V(P^0, g^0, M) > V(P^0, g^1, M - \Omega), \tag{6.1}
\]
\[
U^0 = V(P^0, g^0, M) < V(P^0, g^1, M - \Omega), \tag{6.2}
\]
\[
U^0 = V(P^0, g^0, M) = V(P^0, g^1, M - \Omega). \tag{6.3}
\]

Unlike private good prices (\( P \)) the willingness to pay of individuals is
not observable. Individuals know (if you accept that he has well formed
preferences on the good about which he is surveyed) his reservation price but the analyst does not know it. Expressions (6.1), (6.2) and (6.3) have a random component that allows them to be expressed in terms of probability and therefore to build on the responses a cumulative probability function that allows us to infer the willingness to pay for the environmental good.

To obtain an estimate of the value of $\Omega$ that satisfies (6.3) is the objective of these surveys. Most of them contain the following (Carson et al., 2001):

- An introductory section that contextualizes the project.
- A detailed description of the good on which the interviewee must respond.
- The institutional framework where the environmental good is provided.
- The methods of payment.
- A method to obtain the individual’s preferences with respect to the good.
- Clarification questions that allow us to know why the individual answered some questions as he did.
- A set of questions on the characteristics of the individual, including attitudes and demographic information.

All the above elements require the interviewee to understand exactly what is asked and to be convinced by the information the interviewer has provided before he is asked to answer questions concerning the changes in the environmental good. The survey must be designed so that the individual believes that his responses will influence the decision making process regarding the environmental good. For the exercise to be credible it is necessary for the method of payment to be realistic and for the respondent really to believe he will have to pay the amount stated in his answers if the project is approved.

‘Hypothetical Answers to Hypothetical Questions’

For many economists these design efforts are not sufficient to modify the hypothetical nature of the exercise. The critique of contingent valuation can be summarized in the sentence: ‘We receive hypothetical answers to hypothetical questions.’ Diamond and Hausman (1994) argue that this method of valuation is not valid for measuring the non-use value of environmental goods. Given that we have no real data on transactions in the market for environmental goods for making comparisons with the
Economic valuation of non-marketed goods (II)

responses from respondents about hypothetical willingness to pay, we need standards for judging credibility, reliability and accuracy. The accuracy is improved by increasing the number of responses, but the credibility and the absence of significant bias are not.

Diamond and Hausman (1994, p. 63) argue that the main problem with contingent valuation is not the survey design but that this method does not measure the preferences of individuals:

. . . we do not think that people generally hold views about individual environmental sites (many of which they have never heard of); or that, within the confines of the time available for survey instruments, people will focus successfully on the identification of preferences, to the exclusion of other bases for answering survey questions. This absence of preferences shows up as inconsistency in responses across surveys and implies that the survey responses are not satisfactory bases for policy.

Problems with the contingent valuation method are well known: the hypothetical bias routed in the hypothetical nature of the survey; the strategic bias as the individual intentionally changes its true value to affect the result; the so-called ‘embedding effect’ meaning that when the scope of the environmental good changes the responses do not vary significantly; anchoring bias meaning the influence of the first number provided to the interviewee on his response; and framing bias, which refers to the influence on the way the information and questions affect the individual’s response.

The contingent valuation experts have redesigned their surveys to avoid the most obvious biases, but Diamond and Hausman put the emphasis of their criticism on the fact that the method produces answers that are not consistent with economic theory. They cite the embedding effect, as the willingness to pay values obtained in different surveys are similar regardless of the magnitude of the problem faced by the respondent.

If when asked how much individuals are willing to pay to solve a problem that kills 2000 birds the answer is about the same as when the figure is 20,000 or 200,000 (Desvousges et al., 1993), something worrying is happening, and it may well be explained by the absence of individual preferences with respect to the public good and the failure of the individuals to consider the implications of their answers on their budget constraints.4

If the responses do not measure the intensity of preferences of individuals, the amount they are willing to pay for the environmental good, then what do they measure assuming that these are not random numbers? According to Diamond and Hausman respondents may be expressing an attitude towards the environmental good, expressed on a monetary scale, because this is what has been requested by the interviewer. Maybe
they are receiving some sort of reward for their moral support for a good cause, especially when they do not actually have to pay for it. They may be making a kind of informal cost–benefit analysis on what they consider to be good for the country. They may even be expressing their reaction to the action that has occurred (discharge of oil into the sea) instead of the economic valuation of the change in the environmental good.

Faced with this criticism Carson et al. (2001) point out that consumer sovereignty is an essential principle in economic theory and then it does not make any difference if the willingness to pay for the non-marketed good is motivated by moral satisfaction. Economic theory does not go into analysing the reasons behind the utility obtained when the individual consumes the goods he chooses. Nevertheless the motive is not irrelevant if moral satisfaction does not arise from contributing to the continued existence of an environmental good or raising its quality, for example, but to please the interviewer. The problem now is that the willingness to pay revealed does not reflect any connection to the good, but to the interaction with the interviewer. This effect is avoided by competent interviewers and careful design of the survey.

Regarding the insensitivity of responses to the size or scope of the environmental good, there is empirical evidence that supports the critical position on the irrelevance of the size or scale of the good being valued (2000, 20,000 or 200,000 birds), and also against those who argue that this is a general problem of contingent valuation. Carson et al. (2001) discuss the available evidence holding that, besides the hypothesis of insensitivity being rejected in most studies, the identified problems are associated with poor design of the survey and how it is carried out, problems that have disappeared in contingent valuation surveys that are performed nowadays.5

Another problem is the information handled by the individual about the phenomenon under analysis when responding to the survey. The individual may be interested in preserving some species of birds but be unaware of key biological facts, like the relationship between population size and the probability of survival, as well as the impact of the ecological damage under evaluation with respect to the size variation in the population of that species.

According to Diamond and Hausman these preferences based on limited information are a bad basis for environmental policy, so it would be preferable to have an expert assessment on the impact of the environmental change instead of asking the public directly. For those who believe that the economic valuation of non-marketed goods is possible and its validity depends on a serious effort of the survey design, any relevant information (like the critical population for the survival of some species), should be presented to the interviewee in order to obtain meaningful responses.
Economic valuation of non-marketed goods (II)

Probably part of the discredit of the contingent valuation method is a result of work carried out by many of its proponents. There is no doubt that to devise a hypothetical market, to convince the respondent to answer honestly regarding the valuation of something not directly experienced and to obtain reliable answers is a difficult task. In the words of Hanemann, one of the most prominent economists in the theory and practice of contingent valuation:

. . . it would be misleading for me to suggest that contingent valuation surveys can be made to work well in all circumstances. I am sure situations could exist where a contingent valuation researcher might be unable to devise a plausible scenario for the item of interest. Nor would I wish to argue that all contingent valuation surveys are of high quality. The method, though simple in its directness, is in fact difficult to implement without falling into various types of design problems that require effort, skill and imagination to resolve. Each particular study needs to be scrutinized carefully. But the same is true of any empirical study.

While I believe in the feasibility of using contingent valuation to measure people’s value for the environment, I do not mean to advocate a narrow benefit–cost analysis for all environmental policy decisions, nor to suggest that everything can or should be quantified. There will be cases where the information is inadequate, the uncertainties too great, or the consequences too profound or too complex to be reduced to a single number. (1994, p. 38)

6.3 CONJOINT ANALYSIS

The conceptual basis of conjoint analysis rests on the modelling of the demand of a good as the demand of a given bundle of attributes (Lancaster, 1966). Any good can be seen as a set of characteristics (e.g. a car consists of a given engine, accessories, size, safety devices, design and price). The consumer’s preferences are defined over the set of attributes and it is possible to estimate the values of the characteristics asking the individual to choose between different alternatives consisting of bundles containing different levels of the attributes (e.g. in one option, the attribute spaciousness improves but speed is lower, in another speed and price are higher and so on).

This valuation method has been widely used in marketing and in transport economics. Public transport has been analysed by means of this approach (Domenich and McFadden, 1975). Public transport users value travel time, waiting time, access and egress time, reliability and price, so conjoint analysis offers the practitioner the possibility of estimating how much the individuals value the different characteristics of the service and therefore to allow the economic valuation of different programmes of spending to improve public transport.
There are different methodologies for the practical application of conjoint analysis (choice experiments, contingent ranking, contingent rating and paired comparison) but the basics consists of the description of the good according to the selected characteristics and the levels of these characteristics. The individual is asked to rank, rate or choose several options (differentiated by their characteristics and levels). Choosing or ranking is considered better than rating because respondents are not forced to translate the intensity of their preferences into numbers. The advantage of this approach is that by including price or cost in the options, the trade-offs stated by the respondents allow us to estimate the monetary valuation of different attributes.

Pearce et al. (2006) provide an example of conjoint analysis applied to the clean-up of the river Thames. The objective is to evaluate several investment options to reduce the amount of storm water (sewage litter) entering the river and degrading water quality. The investment costs are known and so we need the economic valuation of improving this non-marketed good. This environmental good is defined as the reduction of sewage overflow entering the river and its attributes are: decreased visual disamenity, fewer days when the exposure to the river water is a health risk, and fewer fish deaths.

The respondent has to choose between the different options, where the level of attributes varies, including the clean-up cost. The utility of individual $i$ is assumed to be a linear function of the attributes (and its levels) in the $j$ different alternatives the respondent is presented and some unobserved factors.

$$U_{ij} = b_1 (\text{sewage})_{ij} + b_2 (\text{health})_{ij} + b_3 (\text{fish})_{ij} + b_4 (\text{cost})_{ij} + e_{ij}, \quad (6.4)$$

where $\text{sewage}$ is the proportion of sewage litter in the river; $\text{health}$ is the number of days per year when water sports are not advisable because of increased health risk (minor illness); $\text{fish}$ is the number of significant fish deaths per year; $\text{cost}$ is the cost of an option (for the respondent). The coefficients $b_1, b_2, b_3, b_4$ are unknown parameters, and $e_{ij}$ is an error term to account for unobservable characteristics.

The estimated coefficients in equation (6.4) are negative, as an increase of any part of this attribute reduces the respondent’s utility. Each coefficient shows the weight an average household places on the corresponding attribute. This is the marginal utility with respect to an attribute; for example, $b_1$ is the change in total utility with respect to a small change in the proportion of sewage litter in the river.

The ratio of one coefficient to another measures the marginal rate of substitution between two characteristics. In this study the estimated
coefficients are $b_1 = -0.035; b_2 = -0.007; b_3 = -0.029; b_4 = -0.019$. The ratio $b_3/b_2$, for example, indicates that the average household values the fish-kills roughly four times as bad as the increase in exposure to minor health risks.

From equation (6.4) it is possible to obtain an average measure of the willingness to pay of the households in the sample (1214 Thames Water customers). The ratio of any coefficient with respect to the cost coefficient indicates the willingness to pay to reduce the value of the corresponding variable. In this study the average household is willing to pay $b_1/b_4$ for a reduction of the attribute sewage, and the value of this ratio is £1.84; $b_2/b_4$ for a reduction of the exposure to risk represented by the attribute health (£0.37); and $b_3/b_4$ for a reduction in the fish deaths represented by the attribute fish (£1.53).

From these implicit prices of the changes in the attributes it is possible to calculate the total benefits of an investment project that reduces the storm water (sewage litter) entering the river. Taking the status quo as the base case characterized by the physical value for each attribute without the project, once we know the expected physical values with the project, the total willingness to pay of the average household to eliminate storm water overflows is:

$$WTP = 1.84\Delta_{\text{sewage}} + 0.37\Delta_{\text{health}} + 1.53\Delta_{\text{fish}}, \quad (6.5)$$

where $WTP$ is the average willingness to pay of each household. Multiplying this value (£76) by the 5.6 million households in the Thames Water area we obtain an estimation of the total willingness to pay per year.

It has been argued that this method is superior to contingent valuation as it is possible to estimate the values of the main characteristics of the environmental good under evaluation. Besides, respondents (in choice and rank modelling) do not have to put a monetary value on his preferences, though a number is obtained through the information revealed in the trade-offs when choosing between options where a cost variable has been included.

Although conjoint analysis can be considered a better option when the environmental good is complex and has multiple attributes, it is true that this complexity creates problems of validity and reliability common to any stated preference method. The cognitive difficulties of the respondent facing choices between bundles with several attributes and several levels for each attribute are quite obvious in creating problems of design and interpretation (see Adamowicz et al., 1998; Pearce et al., 2006).
6.4 INDIVIDUAL PREFERENCES AND SOCIAL WELFARE

Altruism and Non-Use Value

It is argued that individuals are willing to pay to increase the welfare of others besides themselves and, therefore, that the altruistic motive should be added to the purely selfish valuation. If so the value of preserving the environment would be higher than the valuation that does not incorporate altruism. The characteristics of the environmental good favour the individual being willing to pay more for their conservation if he cares about the welfare of others.

Although the results in stated preferences based surveys suggest that people are willing to pay in excess of their private values, it is important to consider the nature of these altruistic feelings in order to answer the question on whether the additional willingness to pay of individual \( i \), because of concern with respect to individual \( j \), should be included in cost–benefit analysis.

With non-paternalistic altruism the utility function of individual \( i \) includes the utility of individual \( j \) as an argument. This means that individual \( i \) cares about the well-being of \( j \) but the consumption bundles of \( j \) are irrelevant for \( i \). In the case of paternalistic altruism the consumption of the environmental good, or the safety increase, by individual \( j \) is an argument in the utility function of individual \( i \).

It is considered that when the altruism is non-paternalistic the altruistic feeling is irrelevant to cost–benefit analysis. The following example (Bergstrom, 2006) serves to illustrate that it is easy to make a significant bias in the valuation if the right question is not formulated. Suppose \( A \) and \( B \) are a couple who maintain separate budgets and are wondering whether to rent a larger apartment with two extra rooms. One room will be exclusively used as a study by \( A \) and the other as a playroom for \( B \) exclusively. \( A \) is willing to pay $100 for the study and, as she loves \( B \), she would pay $50 more to see \( B \) happy with his playroom (which \( A \) will never use). Similarly \( B \) is willing to pay $100 for his playroom and, as he loves \( A \), he would pay $50 more for \( A \) to have her study (which \( B \) will never use).

The additional rent for the new apartment is $250, which will be divided equally. Should they rent it? If the altruistic willingness to pay is included the answer is yes (300 > 250) and if it is excluded the answer is no (200 < 250). What is the correct answer?

Consider the case where the apartment is rented on the basis of inclusion of the willingness to pay for the welfare of the other in addition to one’s own welfare. Once the apartment is rented individual \( A \) is worse off...
because she values the change at $100 and must pay $125 (half the additional rent). However, as she cares about $B$, she reviews the status of her partner and notes that he is also worse off (he paid $125 for something that brings a satisfaction of $100). They were wrong to rent the larger apartment.

The paradox of this result is solved by considering that if the benefits are included, the costs should also be included. Each of the individuals is better off if the other has a good that makes the other happy, but they are also worse off if their partner has to pay a higher rent. The problem is solved in practice by including the costs in the question, or directly excluding altruism. Both alternatives lead to the same result.

It is argued that the willingness to pay of the individuals who care for the well-being of others (e.g. their safety, the consumption of environmental goods) should be included in cost–benefit analysis when the altruism is paternalistic. Then, as important as the intensity of the altruistic preferences is the nature of these preferences. For example, in the case of projects saving lives, it is important to consider how the value of a statistical life has been estimated. Jones-Lee (1992) has shown that with pure paternalism, the value of a statistical life increases significantly (10 to 40 per cent larger in the case of the UK) than the value for a society of purely self-interested individuals.

**Moral Principles and Economic Valuation**

The economic valuation of environmental impacts has also been criticized on ethical or moral principles, with the basic position that the passive use value cannot be measured in monetary terms. Within the passive use value is the existence value. Using willingness to pay, for example, to determine that a species persists is unreasonable from this ethical position. Abortion is moral or immoral and it is still one thing or the other regardless of the amounts that individuals are willing to pay to allow or prohibit it; and it is still moral or immoral at different points in time even if one of the fronts benefits from higher income and declares a higher willingness to pay.

Assuming that the problems of design and implementation of the survey have been solved, contingent valuation has limitations that, although common to the neoclassical measurement of the value of goods, should not be overlooked because of the impact they have on the valuation of unique environmental goods. The first is the acceptance that the willingness to pay is the value of the goods to the society, taking into account that the willingness to pay not only reflects the preferences of the individuals interviewed but also their income. The second limitation is that the preferences of future generations do not count. Only the preferences of the
present generation with respect to itself and to future generations matter, which in terms of the environmental goods legacy to new generations creates an ethical problem.

**Distorted Preferences**

There is another problem that particularly affects the environment and health issues, and it is the distinction between the satisfaction of individual preferences and individual welfare. Adler and Posner (2001) call attention to the uninformed preferences, distinguishing between instrumental, intrinsic, adaptive and objectively bad. Instrumental preferences are changed by providing information: for example refusal to drink water with fluoride because the individual believes that fluoridation causes cancer. The intrinsic preferences change too, but not because there is an error in the individual’s beliefs but because the individual does not have enough information when asked and such information changes the preferences: the willingness to pay for an art project is low because the individual does not know the aesthetic qualities of the project.

The adaptive and the objectively bad preferences create an obvious conflict with the principle of consumer sovereignty. The individual lives in miserable conditions and his preferences have adapted to this situation of precariousness and loss of self-esteem so that, according to those adaptive preferences, he negatively value, for example, an improvement in his environment because his misfortune and social resentment make him believe that the improvement is bad for him. Regarding the objectively bad preferences, again they violate the principle of consumer sovereignty, but if society accepts that there is no social benefit from children smoking, it could give zero value to the loss of profits of the tobacco industry if a project reduces the number of children who smoke. In any case, these are extreme examples and situations that do not affect most projects, but with regard to environmental impacts such as global warming it can be useful to consider the previous arguments about the sometimes complex relationship between individual preferences and social welfare.

### 6.5 THE VALUE OF LIFE

There are projects and policies whose main objective is to reduce physical risk. Other public initiatives, regulations or investment projects, involve an increase or decrease in injuries and deaths, either in the construction phase or during the life of those actions.

The title of this section may seem immoral; one should hasten to make
clear that we are not talking about the life of a particular individual. Human life generally has an infinite value when it comes to one’s own life; this is also so if we are asked to value the lives of our loved ones, and even, in extreme circumstances, many people risk their lives to save a stranger. Apparently neither does the society set limits to saving the life of any given individual. A mountaineer in danger of death will receive all kinds of assistance from the government in order to rescue him, and rescue efforts will not be paralysed because they have reached a certain level of spending.

Nevertheless economists argue that the scarcity of resources makes choices unavoidable and those choices also affect safety. Let us think for a moment what would happen to the number of fatal accidents if it were forbidden to drive at more than 30 km per hour. It is more than likely that the number of deaths in road accidents would be reduced drastically; however, if we ask individuals if they would accept this prohibition in exchange for a reduced number of deaths, in which they or their families have a low but certain probability of being involved, almost certainly a majority would answer no. The reason: the higher cost of travel, with all the inconveniences and economic losses associated with such a reduction in the speed limit.

The above example is deliberately extreme, but it serves to illustrate that individuals trade comfort, speed and income for physical risk, sometimes uninformed, but at many other times they are aware of the risk they bear in exchange for other goods. When one accepts this implicit trade-off we enter the field of the economic valuation of life. The meaning of the valuation is as follows: to the extent that society accepts risks that could be reduced by giving up other public and private goods, it is interesting to see how much individuals are willing to make this transaction. The term value of life acquires a less dramatic and more practical dimension, because what is at stake is not really the value of life in the strict sense but how much individuals are willing to forgo other goods in exchange for living with higher levels of safety, and vice versa.

We will see that in reality what is valued in cost–benefit analysis is the increase or decrease in the likelihood of accidents that can result in injuries or deaths. We are referring to the increase in physical danger through fact of undertaking an activity, like building a bridge or driving a vehicle. The role of economics in this context is that resources are scarce and the need to choose among alternative uses also counts in decisions that include the good safety.

Suppose we are considering building a dam and among its costs we include the probable loss of five anonymous lives during the lifetime of the project. Should the dam be built? If the value of life were infinite, this
project would never be carried out; however it is more than probable that if the benefits in terms of electricity production, for example, sufficiently outweigh the costs of constructing, maintaining and operating the dam, the project will be carried out. If one takes the extreme position of not giving a specific value to the anonymous lives that will be lost, we will be giving them a zero value in the economic evaluation of the project.

Another utility derived from having a figure that is used to quantify in monetary terms the cost of accidents affects the ranking of projects within a limited budget: suppose that the value of a statistical life is estimated at $3 million and we are choosing several projects within a limited budget. One project involves a road that saves time and has the additional benefit of removing a level crossing. It is very expensive, but its implementation is expected to prevent ten deaths per year. The mere inclusion of an additional $30 million per year from the elimination of the level crossing could situate this project above the rest in the ranking.

Finally, it is worth making a distinction between the value of life and the value of a statistical life. In the former case we have a single individual; in the latter case we have a cohort of population of individuals. The value of life is simply the marginal rate of substitution between risk and income. The value of a statistical life is the average of the marginal rate of substitution between risk and income in the cohort.

The value of a statistical life is also estimated through survey-based techniques such as the contingent valuation method and nowadays more commonly through choice experiments (see sections 6.2 and 6.3). The problems associated with the values obtained by surveys in which individuals are asked for their willingness to pay for or accept changes in safety levels are similar to those previously described in the valuation of environmental goods.

Similarly to the environmental good, the revealed preference approach has also been followed to estimate the value of a statistical life. The compensating wage differential is a method based on the identification of contexts in which the individual trades off income against changes in the probability of death or injury. Previous to the analysis of this method we describe the human capital approach based on lost earnings as a result of the death of the individual, which is used for determining compensation in wrongful death settlements.

**Human Capital**

A valuation method used to estimate the value of life is to calculate the present value of the expected future earnings of the victim, given life expectancy and expected income. In legal disputes, for establishing death
compensations in lawsuits, it is common to employ the discounted value of future earnings. A lawyer would argue that the death of a highly qualified professional should be compensated to the value of a sum equivalent to the wages that he had earned during his life.

The value of the discounted earnings \( (DE) \) lost as a result of a death is the discounted flow of annual wages of the victim for the rest of the years that he would have lived if the accident had not happened. Suppose we are estimating the value of the earnings lost as a result of a project in 2010 that causes the death of an individual in that year. One way to calculate it is shown in expression (6.6) where \( w \) is the wage of each year and \( \pi \) is the probability in the base year that the individual is still alive in a given year (denoted by the superscript). In expression (6.7) the length of the series does not matter provided that we do not exclude the years in which \( \pi \) is significantly different from zero.

\[
DE = w_{2010} \pi_{2010} + \frac{w_{2011} \pi_{2011}}{(1 + i)} + \ldots + \frac{w_{2100} \pi_{2100}}{(1 + i)^{90}}. \tag{6.6}
\]

In general,

\[
DE = \sum_{t=t_0}^\infty w_t \pi_t (1 + i)^{-(t-t_0)}, \tag{6.7}
\]

where \( t_0 \) is the base year.

The weakness of this method of valuation is that it uses labour market information in a wrong sense. The value of earnings lost represents the marginal product of the worker to the employer; it is the economic value for the company but not for the individual. Furthermore, taking the argument to the extreme, it could be argued that the death of a pensioner, or a permanently unemployed person, does not imply any social loss.

**Compensating Wage Differential**

This method is based on revealed preferences and it also uses labour market information. The idea is to study the behaviour of wages at different levels of risk. If the individual accepts an additional death risk of 1 in 10,000 when he accepts a particular job, and demands an annual wage compensation of $300, he would be implicitly valuing his life at $3 million.

If we had this information for a sufficiently large number of individuals exposed to different levels of risk that they are aware of, and with different wage premiums to compensate for taking such risks, an estimate of the value of a statistical life could be obtained.
In practice, and from a representative sample of the workforce, this method tries to determine the relationship between the wage gap in different professions with different known probabilities of death, but the required assumptions that workers voluntarily choose those professions and that they are fully aware of the probabilities of losing their lives is not always true.

The estimation of a simple model as in (6.8) would yield $b_1$, the value of a statistical life, by isolating in the coefficient $b_1$ the effects of other variables such as sex, experience, age and a set of relevant elements of utility or disutility of the job.\footnote{A key question with respect to the values obtained in the labour market is whether these values of a statistical life can be extrapolated to the rest of the population. Can we use the value of a statistical life obtained in the labour market to measure the value of life in a project where the affected individuals have characteristics far different from workers of the sample? In general, the value of a statistical life represented by $b_1$ should be taken as a lower bound when extrapolated to the whole population. Moreover, there is not a single value of life even for the same individual. The wage compensation that individual $A$ requires depends on the level of safety in which he is situated in the labour market. It may well be that $b_1$ has been estimated in relatively safe conditions because of strict labour regulations, which means that the compensation that the worker requires is influenced by the ‘high’ safety level in which he is located. Had the same worker negotiated the compensation situated in a lower level of safety, he would have required a higher wage: the marginal rate of substitution between income and safety has changed with the level of safety. Furthermore it is assumed that the worker is a well-informed individual who clearly perceives the probabilities of death in the different jobs being offered, and that he negotiates on an equal footing with the employer when setting his wage. In the real world these conditions are very easily violated. A worker born in a mining area with high unemployment may have no other job alternatives and therefore the risk premium received is not the result of a free negotiation. The consequence of this fact is the underestimation of the value of a statistical life.}

\begin{equation}
\text{Annual wage} = b_0 + b_1 \text{ risk of death in the year} \\
+ \sum_{i=2}^{n} b_i \text{ other relevant variables} + e,
\end{equation}

where $e$ is an error term to account for the influence of unobserved variables.
in the labour market to the rest of the population; that is, for projects involving deaths from pollution, road accidents and other reasons that have nothing to do with the labour market where the value was estimated, let us consider the highly probable case of a downward bias in the valuation of a statistical life. Suppose the case of two different individuals: the worker \( A \) and the individual \( B \), who is more risk averse than \( A \). Both \( A \) and \( B \) are willing to accept higher risks if they are compensated with income; however the compensation required by \( B \) is higher than that required by \( A \). If one accepts that individuals who accept hazardous jobs have, at equivalent levels of risk, a risk attitude less conservative than the rest of the population, the compensation for \( A \) underestimates what \( B \) would require.

Another problem is to use the value of a statistical life obtained in the labour market for projects in which deaths will occur among workers in the sample because of a risk of double counting. If workers have been compensated by raising their wages in free negotiations and with full information of the risks assumed, it is wrong to include separately the cost of accidents by the procedure of multiplying the value of life by the number of estimated deaths resulting from the implementation of the project.

To conclude, it should be emphasized that what is looked for in cost–benefit analysis is that the loss of lives resulting from the implementation of a project, or the lives saved thanks to a project, receive adequate treatment in the calculation of the social profitability of implementing such projects. Maintaining a radical position against that valuation can simply imply that the lives lost, or deaths avoided, receive a value of zero. Economists do not try to calculate ‘the value of life’ in a literal sense, but to approximate the implicit value of the society on another inescapable trade-off: how much of the scarce resources the individuals are willing to sacrifice to reduce risk in their lives and therefore to reduce the number of injured and dead.

### 6.6 BENEFITS TRANSFER

A practical problem in the economic valuation of non-marketed goods concerns the transferability of the results. In practice, given the impossibility of performing specific studies in the evaluation of some projects, practitioners often resort to the use of values obtained in other studies (and other contexts). The popularity of this procedure requires understanding what we are doing when transferring values from other studies.

The main difficulty with respect to the transfer of values of environmental impacts of existing studies is that such values have been obtained in specific contexts that may be hardly similar to the context of the project to
be transferred. It is very important to know exactly what the interviewee was asked when we received his monetary valuation of a given impact. A clear example of the risk of transferring values without prior consideration about how the information was obtained is the following (Johansson, 1993).

Suppose an individual is asked in a survey about his willingness to pay for a Jaguar; in a second study he is asked how much he is willing to pay for a Mercedes; and in the third study for a Volvo. It seems unreasonable to add the three values to obtain the total willingness to pay of this individual for cars. There are two correct and alternative ways to proceed. The first is to ask in sequence about the willingness to pay for a car conditional on the money already spent on the other two cars. The second is to ask directly for the total willingness to pay for the three cars.

As noted by Johansson, the transfer of values from other existing studies to another project is often equivalent to the initial questions about the willingness to pay for each car, in the sense that we are actually adding willingness to pay that has not been conveniently conditioned.

The intuition of the previous example is that if we add unconditional willingness to pay from different studies we may be overestimating the economic value of an environmental good. However the opposite may also occur and we can underestimate a negative environmental impact by adding valuations from studies in a different context.

The mentioned undervaluation can be illustrated with a project involving the purchase of an expensive protection system that prevents oil spills on a coastline that has many beaches. If we add the willingness to pay to protect each separate beach we will be underestimating the value that users attach to the elimination of the risk of contamination.

The reason for the underestimation is that, by adding willingness to pay to keep each beach clean, an individual knows he has any of the other beaches as substitutes and therefore his monetary valuation of the beach he visits will be less than if you ask him how much he is willing to pay to preserve the current state of the beach conditional on the other beaches also being contaminated. To calculate the total willingness to pay we would have to proceed as in the example of cars, resulting in this case in a greater willingness to pay for preserving the beaches given that we have conditioned the question on the fact that the other beaches have also been contaminated.

It is common practice to use values of other studies when budget constraints do not allow conducting a specific survey or perhaps as a first step before a proper estimation of the local values is carried out. In any case, it is advisable to avoid a mechanical transfer of values from other contexts without a previous correction according with some characteristics of the population in site as per capita income.
Sometimes a statistical analysis of a number of studies (meta-analysis) is available, relating the average willingness to pay for a particular non-market good to a set of economic and demographic variables. In this case it is possible to improve the process compared with the unadjusted transfer of values that ignore key differences between the site where the value comes and the context where the value is transferred.

An example of a meta-analysis of the value of a statistical life can be found in Miller (2000). A useful result in this study, for an adjusted transfer, is that the elasticity of the change in the estimated value with respect to changes in income per capita is equal to one, hence the value of life changes by the same percentage that output per capita changes. Miller suggests that the results obtained allow us to transfer these values to other countries. The values of a statistical life in this study show that multiplying per capita income by 120 we obtain an approximate value of a statistical life for the chosen country, except for those countries whose income per capita is below $2000.

**THINGS TO REMEMBER**

- The stated preference approach tries to elicit users’ values through survey-based methods. Contingent valuation or conjoint analysis are two techniques based on the construction of a hypothetical market in which the respondents reveal their willingness to pay for a change in an environmental good or in level of physical risk.

- These methods are an alternative to the revealed preference approach when it is not possible to observe users’ behaviour in a related market. This is the case of the non-use value, or passive value, of environmental goods, which requires the construction of hypothetical situations and whose results are very sensitive to the design of the survey. Some critics argue that the main problem is not accuracy but credibility, the fact that the responses obtained in these surveys do not measure the intensity of preferences of individuals. Important research effort has been made to improve the methods and avoid the most obvious biases.

- Conjoint analysis has been presented as an alternative to contingent valuation as the respondent only has to choose between options and indirectly reveal his willingness to pay for the attributes of the non-marketed good. Conjoint analysis is particularly appropriate when the good has multiple attributes but this also has its drawbacks in terms of the increased complexity of the exercise and its corresponding cognitive difficulties for the interviewee.
Changes in physical risk, such as the reduction of the probability of severe injury or death, is the objective or a side effect of many projects. Practitioners conducting cost–benefit analysis of regulatory proposals and investment projects that prevent, or increase, the number and seriousness of accidents, need to assign an economic value to these effects. The aim is to include them in the calculation of the social profitability of such projects. Maintaining a radical position against that valuation can simply imply that the lives lost, or deaths avoided, receive a value of zero.

A common practice in cost–benefit analysis is to transfer values obtained in other studies to avoid the costs of conducting specific surveys. It is important to emphasize the fact that the values of a statistical life or environmental impacts of existing studies are context specific and perhaps not comparable to the context of the project to be transferred. A prerequisite to the transferability of values is to know exactly what the interviewee was asked when he provided his monetary valuation of a given impact.

NOTES

2. The report is signed by Kenneth Arrow, Robert Solow, Raul R. Portney, Edward E. Leamer, Roy Radner and Howard Schuman.
3. The elicitation question to obtain \( \Omega \) has been presented in different formats: open-ended, bidding game, payment card, single-bounded dichotomous choice, double-bounded dichotomous choice and one and a half-bounded dichotomous choice (see Pearce et al., 2006).
4. For a review of similar cases in surveys of willingness to pay for public goods, see Frederick and Fischhoff (1998).
5. For a discussion of the problems with contingent valuation see Pearce et al. (2006).
6. For a complete description of the study see EFTEC (2003).
7. Viscusi (1993) reviews 24 studies that estimated the value of life in labour markets. The results were in the range of 3–7 million dollars in 1990. In a study conducted by Miller (2000) the estimated values are taken from the value of a statistical life in 68 studies for 13 different countries. Miller estimates an equation relating the value of a statistical life to the gross national product per capita (1995) in the country. The results obtained by treating the value of each country as a single observation show an average of $3.45 million in 1995 with a coefficient of variation of 65 per cent. We must keep in mind that there is disparity of income per capita among the 13 countries, which include Australia and South Korea, for example.
7. Discounting and decision criteria (I)

This picture, in the large, of society arranging, modifying, adjusting its total income stream as between this year and later years is the most important picture we can draw of investment opportunity not only because it automatically leaves out borrowing and lending, or buying and selling, but also because it automatically reduces the picture of income to its fundamental terms of real or, as I prefer to call it, enjoyment income and its obverse, labor pain. We do not have to think so vividly, as we do in the case of an individual, of money items and intermediate processes. We can without difficulty fix our attention on the final consumption. Society is like Robinson Crusoe picking and eating his berries, however complicated may be the apparatus which intervenes between the labor of picking and the enjoyment of eating.

(Irving Fisher, 1930)

7.1 INTRODUCTION

Benefits and costs occurring in different years have to be aggregated to obtain the net present value ($NPV$) of the project. In cost–benefit analysis, economists add in a single dimension the benefits and costs of different nature. To add units of benefits or costs corresponding to different individuals or to different time periods we require a weighting system. The first weighting would be basically for reasons of equity (see section 2.5), and the second one, covered in this chapter and the next, because the temporal dimension of benefits and costs matters. The average individual prefers (without inflation) a monetary unit today rather than next year.

This chapter covers the mechanics of discounting (sections 7.2 and 7.3). Homogenization involving the arithmetic of discounting allows the comparison of the sacrifice of present consumption implied by the decision to invest in a project with the flow of net benefits that occurs throughout the life of this project. Once the benefits and costs have been discounted we need a decision criterion in order to accept–reject a project and compare between different projects. In section 7.4 we present the most important economic indicator, the $NPV$, to help take those decisions. The $NPV$ allows us to express the whole flow of benefits and costs of the project in a single figure. The internal rate of return and the benefit/cost ratio are also discussed.
7.2 DISCOUNTING THE FUTURE

Most projects involve costs in the present in exchange for a stream of net benefits in the future.¹ The flows of expected benefits and costs resulting from the implementation of the project must be aggregated in order to obtain the $NPV$. This economic indicator represents in a single figure the net benefit of the project, the profitability (social or private) of the project under evaluation.

The basic decision rule in both the financial and the economic evaluation of projects is: accept the project if the $NPV$ is positive and reject it if it is negative,² since in that case we can make better use of the resources in other alternatives. A positive $NPV$ is a necessary condition to undertake a project, but not a sufficient condition, since other projects with a positive $NPV$ could be more socially desirable than the first in a context of limited funding.

Even in the absence of uncertainty, individuals do not generally assign the same value to a monetary unit regardless of the time period in which it is received. Only in cases where individuals are indifferent between present consumption and future consumption could the net benefits of different periods be added unweighted. An expression for the calculus of the $NPV$ that allows for a positive marginal rate of time preference (1 unit is more highly valued at present than in the future) is the following:

$$NPV = \sum_{t=0}^{T} \delta^t (B_t - C_t), \quad (7.1)$$

where:
- $B_t$: benefits in year $t$
- $C_t$: costs in year $t$
- $T$: life of the project
- $\delta$: discount factor (assumed to be constant over time).

The common discount factor used in cost–benefit analysis is shown in expression (7.2). This is what is called exponential discounting, which gives exponentially decreasing weight to the benefits and costs occurring in the future:

$$\delta' = \frac{1}{(1 + i)^t}, \quad (7.2)$$

where $i$ is the social discount rate.

The mechanics of discounting values generated in different time periods is based on the assumption that the individual has a stronger preference for the present than the future. If the individual’s utility depends on
consumption in successive periods we assume that the individual gives more weight to the consumption that is closer to the present, so that his utility function includes a positive marginal rate of time preference that discounts the value of consumption according to its location in time.

There are two opposing forces in the utility function of individuals with regard to the discounting of consumption over time. One supports the idea of not discounting because of the diminishing marginal utility of consumption; the other one goes in the opposite direction and supports discounting because of the impatience of individuals, whatever the reasons for it. The first justifies the individual trying to spread the consumption over different time periods. The second explains that an individual values a consumer unit today more than a consumer unit in the future, valuing it less the more we move away from today.

It can be seen that the discount factor in (7.2) has two fundamental characteristics: it is less than 1 for values of \( i > 0 \) and \( t > 0 \), and it decreases rapidly as \( t \) increases. Its implementation implies reducing the present value of benefits and costs that occur when \( t > 1 \) and also implies that benefits and costs far apart in time are irrelevant.

If the discount rate is, for instance, 6 per cent, a rate in real terms widely used in the economic evaluation of projects in recent years, the value of 100 units of benefit (or cost) in year one is converted into 94 units in the base year. In the fifth year 100 units are equivalent to 75 once they are discounted to the present. In 30 years the 100 units are reduced to 17 in the year zero. In 100 years the present value of 100 is 0.3.

This fact has been of concern to environmental economists, who believe that exponential discounting penalizes projects whose benefits are realized in the long term (e.g. reforestation), and benefits projects with huge costs in the distant future (e.g. radioactive waste). That is, it ignores the welfare of future generations.

As an alternative to exponential discounting, some economists have proposed using a hyperbolic discount factor, which, although it is also less than 1 for values of \( i > 0 \) and \( t > 0 \), its value decreases more slowly when \( t \) increases. Unlike exponential discounting, with hyperbolic discounting the present value of benefits and costs that occur in the distant future does affect the profitability of the project. It has also been argued that when there are different interest rates to reflect individual differences in intertemporal preferences, and the government takes an average of these preferences, the resulting interest rate decreases with time, tending to the lowest value of the range of rates as we move away in time (see section 8.5).

The time horizon of many projects can exceed 40 years or can be considered as infinite for evaluation (e.g. some sections of roads built by the Roman Empire are still in use). The choice of discount rate is one of the
key aspects of economic evaluation as it can dramatically affect the profitability of the project, or alter the selection of projects as it changes the relative profitability of a project in relation to others that are competing for funding. When the flows of net benefits of the projects being compared have different profiles, the discount rate may be decisive in the selection process, as we shall see below.

As noted previously the common approach to discounting is shown in expression (7.2). This discounting procedure is generally accepted for those projects that do not involve a loss of human lives or environmental impacts, or significantly affect the welfare of future generations.

Let us see with an example the arithmetic of discounting. Project A, represented in Figure 7.1, has a duration of 11 years. Initially (base year: \( t = 0 \)) it requires an investment of $2500. After a year the project generates net benefits of $2000. From the second until the eleventh year, the annual net benefits are $100. At the end of the project the residual value is zero.

According to (7.1) the economic profitability of project A is as follows:

\[
NPV(A) = -2500 + \frac{2000}{1 + i} + \sum_{t=2}^{11} \frac{100}{(1 + i)^t}.
\]

The profitability of the project, measured by the \( NPV \), heavily depends on the chosen discount factor. Suppose that the discount rate is 5 per cent. In this case the \( NPV \) of the project is positive \( (NPV(A|i = 0.05) = 140) \), so the discounted net benefits are larger than the initial investment costs. The value of $140 is equal to the investment less the discounting value of the benefits from years 1 to 11. This positive difference of $140 indicates that the project allows the remuneration of the investment at the opportunity cost, recovering the invested capital and generating a social surplus that, valued in year zero, is equal to $140.

Let us maintain the same time profile of project A and the same initial investment, but change the discount rate. For example suppose that the interest rate on this project is 10 per cent instead of 5 per cent. The result of the investment is now negative \( (NPV(A|i = 0.10) = -123) \). The flow of net income represented in Figure 7.1 is not enough to offset an investment that now demands a higher return, raising its opportunity cost.
Figure 7.2 shows how the \( NPV \) of project \( A \) changes when the discount rate changes. In this way, at an interest rate equal to zero, the \( NPV \) is equal to $500, which is the direct sum of the net benefits less the initial investment cost; when the interest rate is 5 per cent the \( NPV \) is equal to $140. The \( NPV \) curve as a function of \( i \) cuts the horizontal axis at an interest rate equal to approximately 7.5 per cent. This interest rate, for which the \( NPV \) is zero, is called the internal rate of return (\( IRR \)) and indicates that the net benefits of the project equal the opportunity costs, valued at 7.5 per cent per year; or, equivalently, the project allows the recovery of the capital invested and the remuneration of the investment at 7.5 per cent per year.

We have seen how the \( NPV \) is reduced when the discount rate rises, becoming negative for interest rates higher than the \( IRR \). Even when it does not change the sign of the \( NPV \), applying a higher discount rate can affect the profitability of various projects with different time profiles, and change the order of preferences among such projects. Consider the case of two projects, \( B \) and \( C \), whose temporal profiles are represented in Figure 7.3.

The economic life of both projects is 11 years. They also require the same investment costs in the base year ($2000). Projects \( B \) and \( C \) differ, however, in the profile of their annual benefits. Project \( B \), in contrast with project \( C \), has an important part of its benefits at the beginning of its life.

The economic returns of both projects are:

\[
NPV(B) = -2000 + \frac{2000}{1 + i} + \sum_{t=2}^{11} \frac{100}{(1 + i)^t} \tag{7.4}
\]
Introduction to cost–benefit analysis

\[ NPV(C) = -2000 + \sum_{t=1}^{9} \frac{100}{(1 + i)^t} + \frac{2000}{(1 + i)^{10}} + \frac{2100}{(1 + i)^{11}} \] (7.5)

It can be appreciated how the value considered for the interest rate affects the two projects unequally. Suppose that initially the interest rate is 5 per cent. Applying this discount rate, project \( C \) is preferred to project \( B \) since \( NPV(C/i = 0.05) = 1166 \) and \( NPV(B/i = 0.05) = 640 \). Both projects are socially desirable when the interest rate is 5 per cent and there are no budget constraints because they have an \( NPV \) greater than zero. However, in cases where there are limited funds ($2000), project \( C \) would be preferred to project \( B \).

What happens with a higher discount rate, for example an interest rate of 10 per cent? The two projects are still profitable, \( NPV(B/i = 0.1) = 377 \) and \( NPV(C/i = 0.1) = 83 \), but now project \( B \) is preferable to project \( C \). It is interesting to examine why the selection of projects changes when we change the interest rate. The rationale is that the profitability of a project depends not only on the magnitude of its benefits, but also on when they occur; that is, their location in time. Without discounting, the net benefits of project \( C \) are higher than those of \( B \); however they occur later in time. A high interest rate ‘penalizes’ these benefits, indicating that they have less present value for the individuals because they occur in the future.

The discount factor acts as a weight for the benefits and costs over the life of the project. This weight tends to zero as \( t \) increases (provided that \( i \) is greater than zero). The economic rationale is clear: if the interest rate applicable to the project rises, the opportunity cost of the initial investment increases and the later the benefit is realized the lower its present value is. Figure 7.4 shows how the ranking of two mutually exclusive projects changes when modifying the interest rates.

As Figure 7.4 shows, the profitability of a project compared with its alternative is sensitive to the interest rate. For interest rates between 0 and
approximately 7.9 per cent, \( C \) is preferred to \( B \); however, for values of \( i > 0.079 \), the comparative advantage of \( C \) disappears. Raising the interest rate has changed the order of preferences of two mutually exclusive projects.

7.3 THE MECHANICS OF DISCOUNTING: SOME USEFUL FORMULAS

Cost–benefit analysis is most frequently applied to physical investment projects that involve some initial years with costs only, and benefits and operating costs when the project is open to public use and until the end of its life. The evaluation of regulatory measures, or any other public policy that can sensibly be quantified in economic terms, has an associated flow of benefits and costs during the \( T \) years that have to be added to obtain the \( NPV \).

Year \( T \) may not be the last year of the life of the project but its duration for evaluation purposes. The reason that \( T \) can be less than the real life of the project may be because the regulatory framework so requires, because it is pertinent to evaluate the period covered by the contract of private participation or because it is expected that, technologically, \( T \) is the maximum length that can reasonably be considered before technical obsolescence applies.

In any of the above cases it is necessary to calculate the residual value of the project. This value is the net present value in year \( T \) of the benefits and costs of the project until infinity. This could mean the cost of dismantling a provisional infrastructure with no further use beyond \( T \); or perhaps the benefits obtained from the future use of the assets and deducting the costs

---

**Figure 7.4** NPV as a function of the interest rate
Introduction to cost–benefit analysis

of obtaining such benefits, which would include the cost of dismantling the equipment or cleaning and decontaminating the soil, and so on. Sometimes the residual value is calculated as a percentage of the investment costs, as an estimate of the remaining value of the assets, but this accounting practice seems to be quite unrelated to the concept of the remaining social value of the project (the benefits and costs beyond $T$).

Leaving aside the investment costs (which we assume for simplicity to correspond to the year zero) and focusing our attention on the flow of benefits and costs from year one onwards, we have:

$$a_T t_5 1 d_5 (Bt_5 2 Ct). \ (7.6)$$

Calling $V_t$ to $(B_t - C_t)$ in expression (7.6) and assuming that $V_t$, realized at the end of each year, is positive and constant ($V_t = V$) during the life of the project, we have the time profile of net benefits shown in Figure 7.5.

The $NPV$ is equal to:

$$\sum_{t=1}^{T} \delta^t (B_t - C_t). \ (7.6)$$

Calling $V_t$ to $(B_t - C_t)$ in expression (7.6) and assuming that $V_t$, realized at the end of each year, is positive and constant ($V_t = V$) during the life of the project, we have the time profile of net benefits shown in Figure 7.5.

The $NPV$ is equal to:

$$\sum_{t=1}^{T} \frac{V}{(1 + i)^t} = \frac{V}{1 + i} + \frac{V}{(1 + i)^2} + \cdots + \frac{V}{(1 + i)^T}. \ (7.7)$$

To calculate the $NPV$ in (7.7) remember that this is the sum of the terms of a geometric progression, whose first term is $a = V/(1 + i)$ and the common ratio is $\delta = 1/(1 + i)$. The sum of this finite geometric progression is equal to:

$$V \left[ \frac{1 - (1 + i)^{-T}}{i} \right]. \ (7.8)$$

Expression (7.8) is the general formulation for calculating the $NPV$ of a stream of a constant value for any duration. The expression in parentheses is the factor that turns a constant flow of benefits over time into a value discounted to the present. This factor increases with the duration of the project and decreases with the interest rate. When $T$ tends to infinity, expression (7.8) is equal to $V/i$.

If the time profile of the project includes benefits in year zero (Figure
7.6) we simply have to add $V$ to expression (7.8). Calculating the sum of the progression of $T + 1$ terms we obtain expression (7.9).

$$NPV = \frac{V(1 + i)}{i} \left(1 - \frac{1}{(1 + i)^T}\right). \quad (7.9)$$

Expressions (7.8) and (7.9) for the $NPV$ rely on the assumption of constant annual benefits. When benefits change over time according to a constant average annual rate, both expressions must be corrected. Suppose that $V$ is a function of GDP and grows at an annual rate equal to $q$, so that $V_t = V_t(1 + \theta)$. Now the time profile is represented by Figure 7.7, and the equation for the $NPV$ is:

$$NPV = \frac{V}{(1 + i)} + \frac{V(1 + \theta)}{(1 + i)^2} + \ldots + \frac{V(1 + \theta)^{T-1}}{(1 + i)^T}. \quad (7.10)$$

If $\theta < i$, (7.10) is equal to:

$$NPV = V \left[\frac{1 - (1 + \theta)(1 + i)^{-T}}{i - \theta}\right]. \quad (7.11)$$

When $T \to \infty$, if $\theta < i$, $(1 + \theta/1 + i)^T \to 0$, and thus for a given $V$ in the first period that grows at a rate $\theta$ to perpetuity:

$$NPV = \frac{V}{i - \theta}. \quad (7.12)$$

The benefits and costs may occur at the end of the year, quarterly or monthly, or almost continuously, as with the demand for a dam that supplies water without interruption or a road that saves time for a continuous flow of vehicles driving on it since opening up to $T$. 

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**Figure 7.6** Benefits in year zero and at the end of the year

**Figure 7.7** Benefits grow at a constant rate
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When benefits are produced twice a year, for example, and the annual discount rate is $i$, we have the time profile shown in Figure 7.8.

To discount the benefits, we need the semester interest rate $i_s$ equivalent to the corresponding annual rate $i$ (or, in general, the interest rate that matches the corresponding subperiod) for the discounting of the benefits that occur every six months (or in other periods shorter than a year) called $V_s$, so we have:

$$(1 + i)^s = (1 + i_s), \quad (7.13)$$

where $s$ is the number of subperiods (semesters in this case). Solving for $i_s$:

$$i_s = (1 + i)^{1/s} - 1. \quad (7.14)$$

Given that $1 + i_s = (1 + i)^{1/s}$, the discounting for the first four semesters in Figure 7.8 is as follows:

$$NPV = \frac{V_s}{(1 + i)^{0.5}} + \frac{V_s}{(1 + i)^1} + \frac{V_s}{(1 + i)^1.5} + \frac{V_s}{(1 + i)^2} \quad (7.15)$$

or

$$NPV = \frac{V_s}{(1 + i_s)} + \frac{V_s}{(1 + i_s)^2} + \frac{V_s}{(1 + i_s)^3} + \frac{V_s}{(1 + i_s)^4}. \quad (7.16)$$

Expression (7.17) generalizes the calculus of the $NPV$ of a stream of benefits $V_s$ that occur in periods shorter than a year. It is obtained by replacing the number of subperiods and the subperiod interest rate obtained according to (7.14) in (7.8):

$$NPV = V_s \left[\frac{1 - (1 + i_s)^{-T_s}}{i_s}\right]. \quad (7.17)$$

The discount formulas obtained above are useful for constant flows of benefits and costs ($V_s = V$) or those that grow at constant rates ($\theta_s = \theta$). In practice this is not often the case, as the annual values do not conform to a pattern that allows the direct application of the previous expressions.\(^5\)

It is interesting to note that many projects have annual benefits $V$ that
are generated in an almost continuous manner, but they are often treated as if they occur at the end of year, as in Figure 7.5. Consider the case of an airport, a dam or a sports centre whose output is produced from 1 January until 31 December. It seems obvious that if the benefits are located at the end of the year, as happens to be the case, the \( NPV \) appears to be lower (the further in time the benefits the lower the present value).

It can be argued that if benefits and costs occur every day of the year, and must be located in an instant of the year, it is more appropriate to place them in the middle of the year than at the beginning or the end. The time profile is shown in Figure 7.9.

The formula for discounting the annual benefits located in the middle of the year, applying the annual discount rate, is as follows:

\[
NPV = V \left[ 1 - \frac{(1 + i)^{-T}}{i} \right] (1 + i)^{0.5}. \tag{7.18}
\]

Expression (7.18) corrects the annual discount formula (7.8) with the factor \((1 + i)^{0.5}\) in order to place the discounted value at time zero, otherwise the application of (7.8) would have located it at time ‘\(-0.5\)’. Applying (7.18) we would obtain the same results as applying (7.17) with benefits expressed per day \( (V_s = V/365) \); that is, the benefits are realized daily and discounted with a daily discount rate that corresponds to the annual rate.

If the project delivers a continuous flow from the beginning of the first year of operation (health service, roads, airports, water supply, etc.), it seems reasonable in view of these results to place \( V \) in the middle of the year, which is equivalent to treating the benefits as if they occur daily or continuously, but applying an annual discount rate.

7.4 DECISION CRITERIA: THE NET PRESENT VALUE

Accept–Reject

There are several alternative ways to measure the social profitability of a project whose benefits and costs occur in different years. The most reliable indicator is the \( NPV \) of the flow of benefits and costs (see Brealey and Myers, 1996). The \( NPV \) summarizes in a single figure the social value of
the project by subtracting the costs \( (C) \) from the benefits \( (B) \), once both have been discounted with the appropriate discount rate \( (i) \). A general expression is as follows:

\[
NPV = B_0 - C_0 + \frac{B_1 - C_1}{1 + i} + \frac{B_2 - C_2}{(1 + i)^2} + \ldots + \frac{B_T - C_T}{(1 + i)^T}. \tag{7.19}
\]

If expression \((7.19)\) is zero, the project’s present value of benefits is equal to the present value of its costs, which would leave the decision maker indifferent between approving or rejecting the project, because implementing the project will only allow the reimbursement of the initial investment and the interest payment, which is what one can obtain by leaving the investment for the next best alternative. This is the starting point for the first decision rule:

\[
NPV > 0 \rightarrow \text{accept the project.}
\]

\[
NPV < 0 \rightarrow \text{reject the project.}
\]

The economic logic of this decision rule is based on the concept of opportunity cost. Consider an investment project of $2486.70 that produces constant benefits equal to $1000 during its three years of life. Suppose the discount rate \( i \) is the bank interest rate (10 per cent) at which this investment would be remunerated if it was not invested in the project. The opportunity cost of this investment expressed at the end of the third year is:

First year: \( C_0 (1 + i) \).

Second year: \( C_0 (1 + i)(1 + i) = C_0 (1 + i)^2 \).

Third year: \( C_0 (1 + i)^2(1 + i) = C_0 (1 + i)^3 \).

An initial figure of $2486.70 reaches the value of $3309.80 in the third year if it is lent to a financial institution. This is the opportunity cost of investing in the project, the renouncing of receiving $3309.80 in the third year because of the project. Therefore, if we invest $2486.70 in the project, we should expect at least $3309.80 in the third year or an equivalent amount during the lifetime of the project, for example $1000 every year over the three years.

When we invest $2486.70 in the project and gain $3309.80 in the third year, the \( NPV \) is:

\[
NPV = -2486.70 + \frac{3309.80}{(1 + 0.1)^3} = 0. \tag{7.20}
\]
The \( NPV \) is zero, indicating that the same payoff is obtained by lending the money to the bank or investing it in the project. The same happens if we gain $1000 each year because:

\[
NPV = -2486.70 + \frac{1000}{(1 + 0.1)} + \frac{1000}{(1 + 0.1)^2} + \frac{1000}{(1 + 0.1)^3} = 0. \tag{7.21}
\]

If by investing $2486.70 in the project we obtain an \( NPV > 0 \), the funds invested in the project generate benefits that exceed the opportunity cost, allowing the recovery of the invested funds plus the interest payment.

When there is uncertainty over the future benefits and costs, the investment is partially or totally irreversible, and it is possible to postpone the investment decision to acquire more information, Dixit and Pindyck (1994) point out that the \( NPV \) rule can be misleading unless it includes the opportunity cost of exercising the option to invest. This investment option is clear in the case of governments that have the property rights for land and natural resources and the competence to decide on the construction of projects of general interest now or later in the future.

In these circumstances of irreversibility, uncertainty and the possibility of postponing, if we invest today we have renounced the value of the information that will be revealed in successive periods, including information that could change the decision to invest, which cannot be altered once the decision to invest has been taken, given the irreversibility of investment and hence the unrecoverable nature of the costs (sunk costs). In the next chapter (section 8.2) we analyse the timing decision, justifying that the \( NPV \) rule is perfectly valid if we include the cost of losing the value of the information that is revealed by waiting.

**Choosing Between Profitable Projects**

The \( NPV \) rule is not only valid for the acceptance or rejection of a project that is evaluated individually. The \( NPV \) is also the appropriate criterion to choose between mutually exclusive projects or to select from a group of projects in a wider range when there is a budget constraint. In this case the objective is to choose the projects that maximize the \( NPV \).

Table 7.1 illustrates the use of the \( NPV \) in contrast with the benefit/cost ratio (\( B/C \)). The \( B/C \) ratio is the fraction of the discounted benefits and the discounted costs, and it is frequently used to present the social profitability of projects. In Table 7.1 four projects are ranked according to their \( B/C \). It can be seen how project \( E \) is the one with the highest \( B/C \) value (1.5) but \( G \) has the highest \( NPV \) value, so the \( B/C \) criterion provides a wrong ranking.
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The lowest benefit per unit of cost corresponds to project $H$, 0.8. One explanation of the rationale for rejecting project $H$ is because $B/C = 0.8$ is less than unity and hence for each monetary unit invested in the project we only recover 0.8, thereby 20 per cent of the investment is lost.

The $NPV$ rule would have rejected project $H$ because it implies losses of $240 if it is undertaken. The criterion of accepting the project with $NPV > 0$ indicates that the profitable projects are $E$, $F$ and $G$, all of them with a $B/C$ greater than 1.

A pervasive characteristic of the real world is the presence of budget constraints; there are more projects with an $NPV > 0$, or $B/C > 1$, than funds available and public agencies are forced to choose between the set of profitable projects, a subset that simultaneously ensures the maximum social welfare and satisfies the budget constraint.

Consider that the budget constraint is $800. In this case we choose projects $E$ and $F$, and the total $NPV$ is $280. The ranking of projects by the indicator $B/C$ leads to the same selection. Suppose now that the budget is extended to $1600. In this case the projects that allow a higher overall $NPV$ ($430$) are $F$ and $G$; and $E$, the one with the higher $B/C$, is outside the subset.

An additional problem with the use of $B/C$ as an indicator of project profitability is that it is sensitive to the way the benefits and costs are computed, unlike the $NPV$, which produces the same result whether the costs are defined as costs or as negative benefits.

Take for example project $E$ of Table 7.1 and suppose that it is composed of two periods – the year zero, in which the investment is materialized, and the year one – and a 10 per cent discount rate. Consider two alternatives compatible with an $NPV$ of $100$.

\[
NPV = -200 + \frac{330}{1 + i} = 100, \tag{7.22}
\]

\[
NPV = -100 + \frac{330 - 110}{1 + i} = 100. \tag{7.23}
\]

<table>
<thead>
<tr>
<th>Project</th>
<th>Benefits</th>
<th>Costs</th>
<th>NPV</th>
<th>Benefits/Costs</th>
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<td>600</td>
<td>180</td>
<td>1.30</td>
</tr>
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<td>960</td>
<td>1200</td>
<td>-240</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 7.1  Indicators for project selection (discounted values in $)
In the first alternative the ratio $B/C$ is equal to 1.5, as shown in Table 7.1. In the second it depends on how we define the costs. If the costs are defined as such, the value of $110$ is a discounted cost with a value of 100, and $B/C$ is equal to 1.5. However, if the cost is treated as a negative benefit (e.g. an aggregation of winners and losers is equal to $220$), the $B/C$ would be equal to two. Note that the $NPV$ is $100$ in both cases.

Although there may be situations in which the $NPV$ is not an immediate criterion, for example when comparing projects of different durations that solve a common problem (see section 8.2), the $NPV$ is widely considered the most reliable criterion, and the only recommendation is to avoid its mechanical use without thinking about the characteristics of the projects that we wish to compare.

**The Internal Rate of Return ($IRR$)**

Another indicator is the $IRR$, which consists of finding the value of $i$ that makes the $NPV$ equal to zero. The $IRR$ is thus the highest discount rate that leaves the project on the border of profitability (see Figure 7.2). The more profitable the project is, the greater the range of values of $i$ with a positive $NPV$.

The decision rule is:

$$IRR > \text{discount rate} \rightarrow \text{accept the project}$$

$$IRR < \text{discount rate} \rightarrow \text{reject the project}$$

Going back to Figure 7.2 it can be seen how the $NPV$ and the interest rate are inversely related. If we conveniently increase $i$, a value of $i^*$ for which the $NPV$ becomes zero is reached ($i^* = 7.5$ per cent). This is the $IRR$ and the decision rule is to accept the project if $i^*$ is higher than the interest rate, and to reject it otherwise.

The $IRR$ also has some problems, since there may be more than one value of $i$ that makes the $NPV$ equal to zero. This can happen if during the life of the project net benefits change the sign. Moreover it is not always the highest $IRR$ project that has the greatest $NPV$.

**THINGS TO REMEMBER**

- Benefits and costs occur at different moments during the lifetime of the project. There are projects with a lifespan of 40 years and others whose effects, in practical terms, can last forever. People are
not indifferent with respect to when benefits and costs happen and usually prefer a unit of benefit today rather than tomorrow.

- If people discount the future we need a method to homogenize the flow of benefits and costs. Exponential discounting is the common procedure to express all the future benefits and costs in present terms. The logic of compounding interest is the rationale behind exponential discounting. If $1 has a value of $(1 + i)$ next year, the present value of $1 next year equals $1/(1 + i)$, where $i$ is the interest rate.

- The net present value of a project is the main economic indicator to express the social value of a project. It consists of the sum of all the discounted benefits and costs of the project and hence the election of a value for the discount rate is crucial in the calculation of the $NPV$. There are other decision criteria such as the benefit/cost ratio or the internal rate of return. The $NPV$ is the most reliable.

NOTES

1. The project life is determined by the evaluating agency. It may or not coincide with the physical life, but it generally coincides with the estimated economic life, which is usually shorter than the physical life. In the case of a longer lifespan than the period chosen for evaluation we need to include the residual value.

2. It should be noted that the basic rule of decision based on the $NPV$ without further qualification requires the investment to be reversible; that is, if the annual benefits are not the expected ones the investment can be recovered; or even if it is irreversible the investment decision cannot be postponed (‘now or never’). If investment is irreversible, there is uncertainty of demand and it is possible to postpone the investment, when calculating the $NPV$ the opportunity cost of killing the option to invest should be included (see section 8.2).

3. Between zero and one, 365 days have passed, so in fact zero and one are moments in time and the year is the interval. For simplicity we refer to the points zero, one, two, etc. in Figure 7.1, and the like, as year zero, year one, year two, etc.

4. Assuming benefits start in year one. This formula allows us, solving for $V$, to calculate the annual value that corresponds to the present value of an asset or a fixed cost that must be annualized: $V = NPV((1 + i)^2)/((1 + i)^7 - 1)$.

5. Spreadsheets have available the $NPV$ and $IRR$ in the menu of formulas.
8. Discounting and decision criteria (II)

Should Americans work harder and invest more to increase industrial production? The economist’s answer is, only if it makes them happier. Newscasters report economic growth as if it were a benefit with no offsetting cost. Growth does benefit individuals, because it allows them to increase their consumption in the future. The conditions that create growth impose costs on individuals, who must work harder and consume less in the present. Is this trade-off worth it? The answer depends solely on the preferences of the individuals themselves.

(Steven E. Landsburg, 1993, p. 101)

8.1 INTRODUCTION

The option accept–reject is the simplest situation we may face when evaluating a project and the sign of the net present value ($NPV$) is enough to make the decision. In other cases we have to choose between different projects, with a positive $NPV$, that are not comparable because of their scale or duration. At other times the $NPV$ is positive, but is higher in an alternative project that consists of delaying the project. In section 8.2 we discuss the decision criteria when one has to choose between mutually exclusive projects with different lifespans and, finally, when the decision can be postponed; that is, it is not a ‘now or never’ proposition, and we need to check the optimal timing.

The social discount rate is a key parameter for the economic evaluation of projects. It determines in many circumstances whether a project is socially worthy, which means that it is desirable to renounce a certain amount of present consumption for the reward of future goods. In section 8.3 we review the concepts of marginal rate of time preference, marginal productivity of capital and interest rate. In perfect capital markets the social discount rate is easy to determine because the three rates are equal. With distortions, like taxes on savings and investment returns, the interest rate is no longer the same as the marginal rate of time preference, and the marginal productivity of capital, hence in section 8.4 we discuss how to calculate the social discount rate in these conditions. Finally, in section 8.5, we address the problem of discounting when future generations are involved.
Introduction to cost–benefit analysis

8.2 DECISION CRITERIA: DIFFERENT LIFESPANS AND OPTIMAL TIMING

Selection Between Mutually Exclusive Projects with Different Lifespans

Using the \( NPV \) as a decision rule, the chance of mistakes in the selection of alternative projects diminishes; however it is important that projects are comparable. An example will help. Table 8.1 shows two mutually exclusive projects with the purpose of saving a natural obstacle in the construction of a road, with the same level of quality broadly defined. Project \( D \) has a lifespan of 50 years, the cost of construction is $1550 and the operation and maintenance costs are $150 per year. Project \( E \) has a shorter life (25 years), its construction costs are $500 and its operation and maintenance costs are $250 per year. Both projects have the same annual benefits ($360), which we assume are limited to time savings. The discount rate is 10 per cent.

There is a trade-off in these projects with the costs of construction and operation: a higher cost of construction at the beginning allows a lower level of operating and maintenance costs during the project life. Calculating the \( NPV \):

\[
NPV(D) = -1550 + \sum_{t=1}^{50} \frac{360 - 150}{(1 + i)^t},
\]

\[
NPV(E) = -500 + \sum_{t=1}^{25} \frac{360 - 250}{(1 + i)^t}.
\]

With a 10 per cent discount rate the values of the \( NPV \) are the following:¹

\[
NPV(D) = -1550 + (9.91481)210 = 532.11,
\]

### Table 8.1 Selection between projects with different lifespans (monetary values in $)

<table>
<thead>
<tr>
<th>Project</th>
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<th>( I )</th>
<th>( C_t )</th>
<th>( B_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>50</td>
<td>1550</td>
<td>150</td>
<td>360</td>
</tr>
<tr>
<td>( E )</td>
<td>25</td>
<td>500</td>
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</tr>
</tbody>
</table>

*Note: \( I \): construction cost, \( C_t \): operation and maintenance annual cost; \( B_t \): annual benefit as a result of time savings.*
\[ \text{NPV}(E) = -500 + (9.07704)110 = 498.47. \] (8.4)

One might think that project $D$ is better than project $E$ because its $\text{NPV}$ is higher; however the projects are not strictly comparable because, while resolving a problem with the same degree of effectiveness, project $D$ does so for 50 years and project $E$ for 25.

The comparison of the two projects requires a previous process of homogenization. Two procedures to make the projects comparable are as follows. The first is based on considering an imaginary project that consists of constructing $E$ twice to resolve the problem for the same time interval as project $D$. From (8.4) we know that the net present value of project $E$ equals $498.47$. An identical project $E$ is constructed at the beginning of year 26 and the flow of net benefits lasts from year 26 to year 50. The $\text{NPV}$ of this identical project is $498.47$ but situated at the end of year 25 (or at the beginning of year 26), so to obtain the present value of the $\text{NPV}$ in year 25 we need to divide by \((1 + i)^{25}\).

\[ \text{NPV}(2E) = 498.47 + \frac{498.47}{(1 + i)^{25}}. \] (8.5)

For a period of 25 years and a 10 per cent discount rate, the discount factor is equal to 0.09230, therefore:

\[ \text{NPV} (2E) = 498.47 + (0.0923)498.47 = 544.48. \] (8.6)

This result shows that it is more profitable to choose alternative $E$, which will be repeated at the end of its useful life.

The other procedure is to calculate the equivalent annual net benefits ($\hat{B}$), which consists of calculating the average annual benefit, which, multiplied by factor $S$ (see Note 1) corresponding to the lifespan and the discount rate, is equal to the $\text{NPV}$ of the project:

\[ \text{NPV} = S\hat{B}. \] (8.7)

Solving for $\hat{B}$, we obtain the equivalent annual net benefit:

\[ \hat{B} = \frac{\text{NPV}}{S}. \] (8.8)

The value of $\hat{B}$ for project $D$ is equal to $53.668$ and for project $E$ is equal to $54.916$, so project $E$ is preferable to project $D$, despite having a smaller $\text{NPV}$.
To Invest or to Wait: When Postponing is Profitable

When the flows of benefits and costs of a project are discounted, and they are compared with the initial investment, we obtain the NPV of the project as a single figure that indicates, if positive, that the project is socially worthy. Claiming that the project has a positive NPV in the base year does not mean that the project should be started. Let us consider two possibilities.

When additional information is not revealed by waiting

Sometimes the comparison between projects consists of checking whether it is more profitable to start today or to wait until next year. There may be many circumstances in a certain world that make it sensible to postpone the project, when this is technically feasible. Perhaps the demand is growing and the social benefits derived from attending to the demand in the first year do not offset the opportunity cost of the required funds. Suppose that the project involves an initial investment equal to $I$ and annual net benefits equal to $B_t$. Postponing the project by one year is profitable if:

$$
\frac{iI}{1 + i} + \frac{B_{T+1}}{(1 + i)^{T+1}} > \frac{B_1}{1 + i}.
$$  \hspace{1cm} (8.9)

The strict inequality in (8.9) reports, in the left hand side, the present value of the benefit of postponing the project by one year, and consists of the interest payment on the investment cost multiplied by the discount actor $\left( \frac{1}{1 + i} \right)$ and the benefits obtained in period $T + 1$ as a result of starting the project a year later. The right hand side represents the discounted lost benefit in year one because of postponing the project.

Assuming that the benefit in the year $T + 1$ is not significant, we can simplify expression (8.9) to:

$$
i > \frac{B_1}{I},
$$  \hspace{1cm} (8.10)

where $B_1/I$ is the rate of return on investment in the first year. If the discount rate is higher than the first year rate of return investing in the project, waiting is preferable.

When delaying the project reveals additional information

When the investment is irreversible, there is uncertainty about the social benefits generated by the project during its lifetime and it is possible
to postpone the project, the alternative of delaying cannot be ignored without affecting the standard \( NPV \) as a decision criterion.

If delaying the project reveals valuable information on the annual net benefits, the economic value of that information is lost when the investment is made. Under the described conditions (i.e. irreversibility, uncertainty and the possibility of delay) there is an opportunity cost of undertaking the investment in the present. Investing today will imply the loss of the economic value of the information revealed by waiting, and therefore it should be included as a cost. The \( NPV \) rule is still valid if the mentioned opportunity cost is included as a cost when calculating the \( NPV \).²

What is the cost of losing this information by investing now? It is the amount the investor (assume it is risk neutral) would be willing to pay to have the option of flexibility. We calculate the \( NPV \) by investing in period zero, then we calculate it again but delaying the project, and the difference is the value that is lost if we do not wait and therefore the cost of investing now.

The above reasoning to calculate the cost of investing now can be presented more intuitively in the cost–benefit analysis framework. The problem could be presented as the decision between two mutually exclusive projects: one consisting of investing now, and the other one delaying the investment. We only have to calculate the \( NPV \) of both projects and choose the one with a higher \( NPV \).

An example may help us to understand the significance of considering the option of waiting as an alternative. The project represented in Figure 8.1 shows an investment in port infrastructure for trans-shipment container traffic.

The project requires an investment of \( $2200 \) in the base year. The social benefits in year one are \( $150 \). In the second year, depending on the decision of a shipping company to sign a long term contract with the port or with a competing port, the benefits go up \( $200 \) or down \( $100 \) with equal probability and forever \( (T \to \infty) \). The decision of the shipping company is known at the beginning of year two. The social rate of discount is 5 per cent.

We face an irreversible investment, there is demand uncertainty and the
investment decision may be postponed to learn the decision of the shipping company. We assume there is no strategic action that may affect the decision of the shipping company, or other additional benefits or social costs than those already described.

The expected $NPV$ of this investment project is as follows:

$$E(NPV) = -2200 + \frac{150}{1+i} + \sum_{t=2}^{\infty} \frac{100 \cdot (0.5) + 200 \cdot (0.5)}{(1+i)^t} = 800. \quad (8.11)$$

As indicated, once the investment has been made and after the first year (with certain benefits of $150), the benefits in the rest of the years may increase to $200 or decrease to $100 with equal probability, depending on the decision of the shipping company.

Therefore, the $NPV$ of the project if the company does not sign the contract, given that the life of the project is infinite, is:

$$NPV_{(no	ext{-}contract)} = -2200 + \frac{150}{1+i} + \sum_{t=2}^{\infty} \frac{100}{(1+i)^t} = -152. \quad (8.12)$$

Alternatively, in the event that the shipping company signs the contract:

$$NPV_{(contract)} = -2200 + \frac{150}{1+i} + \sum_{t=2}^{\infty} \frac{200}{(1+i)^t} = 1752. \quad (8.13)$$

These results show that the investment will lead to a loss of $152 or a benefit of $1752 with the same probability (i.e. 0.5). Therefore the expected $NPV$ is equal to $-152 \cdot (0.5) + 1752 \cdot (0.5) = 800$, a result that we had already obtained in (8.11). If the agent who takes the decision is risk neutral, the project is accepted.

It is worth pointing out that postponing this project by one year would not be profitable if the information revealed in year two is not taken into account, as the benefits of the first year are higher than the interest rate multiplied by the investment (see (8.10)). Therefore apparently there are no gains obtained by waiting. In reality, however, delaying not only implies losing the benefit of the first year and saving the opportunity cost of the money invested (reflected in the discount rate), there is also an additional benefit that comes from the valuable information revealed by waiting.

When the project is delayed by one year we lose the benefits of the first year, but we now know whether the shipping company will sign the contract or not. Therefore we know whether the yearly benefits are $100 or $200 from year two onwards.
Consider an alternative project consisting of not investing in the present and waiting a year. Once the expected $NPV$ has been calculated, compare this value with the expected $NPV$ of investing now, and then choose the option with the highest expected $NPV$. If by waiting we know that the company is not going to sign the contract and therefore the benefits are equal to $100$, the best option is not to invest and because of this decision the negative $NPV$ in (8.12) is avoided, so the $NPV$ is in this case zero.

As this result ($NPV = 0$) can occur with equal probability to the contract being signed and the annual benefits being equal to $200$, the expected $NPV$ of delaying is:

\[
E(NPV)_{\text{delaying}} = \frac{1}{2} \left( \frac{-2200}{1 + i} + \sum_{t=1}^{\infty} \frac{200}{(1 + i)^t} \right) = 857. \quad (8.14)
\]

Comparing the expected $NPV$ of investing now ($800$) with the expected $NPV$ of postponing the investment one year ($857$), the decision is to delay the project, and after waiting a year, build only if the shipping company chooses to operate in the port. The willingness to pay to maintain the option to invest (i.e. to have the flexibility of choosing at the start of the investment) is the difference between (8.14) and (8.11).

It is interesting to note that the expected $NPV$ of investing in the present is positive and therefore the project would be approved if the investment is of the ‘now or never’ type. Even waiting a year would not be profitable when no additional information is revealed. However, if the option to wait is feasible, we may think of it as if there is another project consisting of waiting, that we must compare with the project of investing now. The decision in the previous example is to wait.

The conventional $NPV$ rule is still valid if the issue is addressed as the choice between two mutually exclusive projects – one investing now; another one, waiting one year – since otherwise the mechanical application of the $NPV$ rule would lead to misleading conclusions.

Using the $NPV$ rule as a decision criterion is useful in contexts similar to those described in this example, provided that the cost of investing in the present includes the opportunity cost of not waiting, or that the option to wait is defined as an alternative to investing now, and comparing the $NPV$ of both projects.

Finally it should be emphasized that in the previous discussion we have assumed that the investor is risk neutral. If instead the investor is risk averse, the comparison between investing now and waiting introduces a new element of variability in the outcome that affects the decision: by investing now, the outcome is a loss ($-152$) or benefits ($1752$); by
waiting one year the possibilities are quite different when the project is rejected (zero benefits) or when it is accepted ($1714).³

8.3 THE MARGINAL RATE OF TIME PREFERENCE AND THE MARGINAL PRODUCTIVITY OF CAPITAL

The economic analysis of the intertemporal choice that individuals make between present consumption and future consumption helps us to understand the problem of the choice of the social rate of discount. Although it is possible that the present consumption ($x_0$) and future consumption ($x_1$) are perfect substitutes for some individuals, it is more likely that the trade-offs happen at rates changing according to the endowment of both goods that the individual has at the time of making a choice. The higher $x_0$ is with respect to $x_1$, the more willing the individual will be to give up $x_0$ by an additional unit of $x_1$.

The compensation the individual requires in excess of a unit of future consumption in exchange for giving up a unit of consumption today is called the marginal rate of time preference ($\xi$). Therefore the individual is indifferent between a unit of $x_0$ and $(1 + \xi)$ of $x_1$. However the market compensates the individual with a return that does not necessarily coincide with $\xi$. The rate that indicates what the individual receives is the interest rate ($i$) that represents the additional amount that is obtained on a unit in the future if we give up a unit in the present.

Both $\xi$ and $i$ may be negative: $\xi$ is negative in cases where the individual has a very high initial endowment of present consumption compared with future consumption, and is willing to give up a unit of $x_0$ in exchange, for example, for 0.9 units of $x_1$ ($\xi = -0.1$). It can also be negative in the case of individuals whose future is uncertain in terms of the income they expect to receive or their health status. A negative interest rate is possible if the nominal interest rate is close to the rate of inflation and there are taxes on the interest payment of financial assets.

A third rate in the analysis of intertemporal decisions is the marginal productivity of capital or the internal rate of return on investment ($r$), which indicates what the individual receives as a return if he invests one unit in productive projects. Ignoring the existence of taxes and uncertainty, if $r$ is greater than $i$, investing in a project is attractive because it yields higher returns than putting the money in the bank. Assume that all the projects are ordered from the most to the least profitable. The individual will undertake all the productive investment projects that satisfy $r > i$ until in the last project he is indifferent
between investing in the project and putting the money in the bank \((r = i)\).

An individual who allocates his funds according to this criterion maximizes the present value of his wealth; and once the NPV is the maximum possible, he takes his consumption choices between present and future according to his marginal rate of time preference. If \(\xi < i\) he will lend money to the bank and if \(\xi > i\) he will borrow. In equilibrium the three rates are equal.

The equality \((r = i = \xi)\) means that the marginal rate of time preference is equal to the rate of marginal productivity of capital and the interest rate. This result shows that all the investment projects that offered higher profitability than the individual was willing to sacrifice in terms of present consumption have been carried out. This is so because the existence of a perfect capital market discourages one from investing in his own projects with lower profitability than other investment projects available in the economy, whose profitability is represented by the market interest rate.

An investment project or a public policy means, in many cases, compromising public funds obtained from the private sector with the purpose of financing projects decided by the government that are expected to increase social welfare. The use of these public funds to finance projects has an opportunity cost. Assuming that these projects are funded with taxpayers’ money, they have given up consumption or investment, which was left without funding because the public sector has absorbed those funds.

We have seen how the interest rate, the marginal rate of time preference and the marginal productivity of capital coincide when there are no restrictions on the financial markets, no taxes and no distortions in the production or consumption that prevent the achievement of the equality of the three rates.

Figure 8.2 represents the capital market in an economy without distortions. The interest rate \(i\) is the opportunity cost of capital for the financing of projects. At \(i_0\) the curves of demand and supply of the available funds \((K)\) intersect. The demand curve \(D\) represents the investment options in the economy, investment opportunities with declining marginal productivity. When the interest rate falls, new projects turn out to be profitable in the private sector because the internal rates of return of these projects are now higher than the cost of financing (interest rate).

The supply of available funds \((S)\), savings, increases with the interest rate, indicating that as \(i\) increases, individuals are willing to substitute more future consumption for present consumption. The value of \(S\) at the interest rate \(i_0\) is the marginal rate of time preference; at the interest rate level \(i_0\) savings are equal to \(K_0\), with no more individuals willing to give up present consumption for future consumption at this interest rate.

Let us see what happens with a volume of available funds \(K^*\), where
the marginal rate of time preference ($\xi^*$) is less than the marginal rate of return on capital ($r^*$). In this disequilibrium case there are some projects that produce higher returns than lenders require to give up present consumption. It is socially desirable to transfer funds from lenders to investors, and this is what will happen in the capital market if there are no distortions that prevent it.

For the level of investment $K_0$ there is no investment project whose internal rate of return is less than the rate of time preference. Then it makes no sense to invest in public projects that do not achieve a return equal to $i_0$ (remember that there are no distortions such as externalities or taxes). If the public agency decides to invest in projects with a rate of return lower than $i_0$, the social welfare decreases, as present consumption will be replaced by future consumption at a suboptimal rate. It can be concluded therefore that, under the conditions described above, the marginal rate of time preference equals the social discount rate and the interest rate ($i = \xi = r$).

8.4 THE SOCIAL RATE OF DISCOUNT AND THE RATE OF INTEREST

Figure 8.3 represents a capital market with distortions caused by taxes on savings and corporate profits. The supply curve $S$ shifts upwards, $S^*$ being the function that represents the interest rates required for different amounts of savings. The height of the shift is the tax, which is assumed
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to be constant per unit of returns on savings. Similarly the investment demand function shifts to the left and downward in the amount of the unitary tax on profits, reflecting the fact that the internal rate of return on investment has declined for all the projects.

The new equilibrium interest rate can be equal to (as in Figure 8.3 for convenience), greater than or less than the one in a market without restrictions, depending on the elasticities of supply and demand. The amount of investment $K_b$ is clearly lower than $K_0$, because of the higher cost of borrowing and the lower returns on investment. The social discount rate in a capital market without distortions is equal to the market interest rate because $i = \xi = r$, while in the most realistic case represented in Figure 8.3 $i \neq \xi \neq r$. The question is which of the three rates should be used as the social rate of discount.

If the project is financed with savings of the consumers, the social discount rate is $id$, which is the marginal social rate of time preference, lower than the market interest rate as represented in Figure 8.3; on the contrary, if the project is financed with funds displaced from private investment projects, we have to use the social rate of discount $ie$, higher than the market interest rate, as this marginal rate of return on capital is what is being obtained from the displaced projects in the private sector of the economy.

It is convenient to distinguish two different situations: first when the public sector competes with the private sector to implement a specific investment project (in this case the marginal rate of return on capital

\[ \text{Figure 8.3  Determination of the social rate of discount} \]
should be used as the social rate of discount); and second when we evaluate projects within the public sector, discussed below.

When the project receives funding from several sources, one way to find the social rate of discount is to find a weighted average of the marginal rate of time preference ($\xi$) and the marginal rate of return on capital ($r$). Thus the social rate of discount is equal to $\alpha r + (1 - \alpha)\xi$, where $\alpha$ is the proportion of funds obtained by the displacement of private investment and $(1 - \alpha)$ the proportion of funds that come from shifting consumption. The calculus of the social discount rate as the weighted average of both rates requires information on the source of investment funds (Harberger, 1976).

The economic criticism of this approach leads to a more complicated alternative. The methodology proposed as an alternative to the weighted average of Harberger is the discounting of the flow of benefits and costs using the marginal social rate of time preference as the social discount rate, but having previously converted the flow of net benefits into a flow of consumption, using the shadow price of capital. For example if in the private sector the marginal rate of return on capital before taxes is 20 per cent and the marginal social rate of time preference is 5 per cent, in the margin, an additional investment unit is four times more valuable than a unit of consumption.

The society is investing below the optimum ($K_b < K_0$ in Figure 8.3) and therefore the flow of benefits is corrected depending on its destination (consumption or investment). If funds are reinvested, they will be multiplied by the shadow price (in our example, $r/\xi = 4$) and then they will be discounted with the marginal social rate of time preference. Another way to see the logic of this procedure is as follows: in year $t$ if the benefits $V$ are wholly reinvested, each subsequent year we obtain $rV$ of return in consumption and reinvest $V$, so $rV$ in perpetuity is equal to $rV/\xi$, which will be discounted by multiplying by $1/(1 + \xi)^t$.

This procedure is more complicated because of the information that is required on the destination of the benefits throughout the life of the project. For this reason, in the practice of economic evaluation, practitioners frequently choose more pragmatic approaches consisting of using, for example, the interest rate of long term treasury bonds in the belief that in the private sector no investment will be undertaken with a lower marginal return.

However we must not forget that when there are taxes on profits, marginal rates of return on investment are greater than the interest rate; and in the case of taxes on funds supplied by savers who do not displace private projects, it is possible to find very low marginal rates of time preference.

In this and the previous chapter we have seen the consequences of the
value of the social discount rate in the allocation of consumption between
the present and the future. Based on the empirical evidence,5 a social time
preference rate in the 3–5 per cent range seems appropriate. These rates
are considerably lower than the 6–8 per cent recommended in the recent
past in the EU and the US, and it is convenient to have in mind that these
rates are risk free, so it is assumed that the flows of benefits and costs are
expressed in certain equivalents. In practical terms, a reasonable course
of action may be to work with the lower and upper bounds and to see the
sensitivity of the project’s social profitability.

8.5 INTERGENERATIONAL DISCOUNT

The conversion of future benefits and costs to present values, according
to the traditional procedure based on the logic of compound interest, pro-
duces results in which the distant future does not apply unless the discount
rate is very low. When the future is defined within the coordinates of the
private sector, 20 or 30 years is regarded as the very long term. In the evo-
lution of species, 300 years is an insignificant period.

Exponential discounting is appropriate for projects or policies that
affect the same individuals at different time periods. Many public projects
with a lifespan of 20 or 30 years can be evaluated with the usual discount
approach of the financial analysis. The problem arises when investment
projects or policies that affect the stock of natural resources have a posi-
tive or negative impact on the welfare of future generations.6 Should the
costs and benefits of future generations be discounted?

Applying the method of exponential discount the distant future becomes
irrelevant for relatively low interest rates. The present value (assuming
zero inflation) of benefits (or costs) of $100 million that occur within 300
years is $2.60 if discounted at 6 per cent (a rate frequently used in the
last decade). The benefits of preserving some species or preventing global
warming are very low if they are discounted with the same discount rates
as short term actions. With the above figures, it would not be profitable to
invest $3 today at a 6 per cent discount rate to avoid a loss of $100 million
within 300 years.7

In the case of the energy policy, the choice of discount rate is criti-
cal. If we want to have alternative sources of energy available for future
generations we must begin to investigate this in the present. The research
and development costs are very high and the expected benefits are distant
in time, therefore by applying a high discount rate it may become an
unprofitable R&D policy in sustainable energies (Lind, 1982).

The same argument applies to nuclear power, which produces immediate
benefits (cheap energy) but social costs that may occur hundreds of years later. In this case many find it morally unacceptable to apply a discount rate that renders negligible the costs associated with exposure to radiation that may be suffered by future generations. In contrast, an argument in favour of discounting the benefits of future generations is the possibility of per capita income continuing its past growth rate. If future generations are much wealthier than we are, to what extent is the sacrifice of the present generation justified? Nevertheless, this argument does not seem to apply to serious irreversible damages.

Another view of this issue is that discounting flows of benefits and costs of future generations is fundamentally an ethical problem, and so it might be adequate to separate the discounting of the benefits and costs that occur in different periods of time with issues of intergenerational equity. This position suggests that the problem is not the choice of the value of the discount rate, but the state of the stock of natural resources that we must leave in the best possible conditions to future generations. In this view the debate over the use of natural resources would take place in the context of sustainable development (Heal, 1997).

Criticism of exponential discounting in projects with very long term effects is not exclusively based on ethical arguments. The evidence on the preferences of individuals with regard to the trade-off between present and future consumption shows the existence of a wide range of discount rates. Two interesting lines of work are those described below. Both are based on the preferences stated by individuals when they are interviewed about their intertemporal preferences in the long term. Both lead to the conclusion that future benefits and costs should count.

Based on the stated preferences in interviews with 3200 households with respect to the implicit discounting of deaths avoided at various future dates, Cropper et al. (1992) concluded that, although the respondents give more value to lives saved in the present than in the future, the implied discount rate in accordance with their responses is not constant. Instead of the exponential discounting, a hyperbolic discount factor fits the preferences stated by individuals subject to the dilemma between lives saved in the present and in the future. The results of the work of Cropper et al. are:

- A high proportion of respondents would not accept life-saving programmes in the future at the expense of giving up programmes that save lives in the present, even if the number of lives saved in the future is 50 times larger. The reason why many of the respondents always preferred to save lives in the present lies in the belief that society would find other means of saving those lives in the future (hence they did not
accept the alternative suggested in the survey). The other reasons are because this will protect them and their loved ones, and programmes of the future (especially those in 100 years time) will not.

- The implicit discount rates obtained are significantly higher than zero, even at horizons as far as 100 years. Although individuals discounted the future, they did not do so at a constant exponential discount rate. The discount rates are much higher for short time horizons than for distant horizons, and there is considerable heterogeneity of discount rates. The standard deviation of the distribution of discount rates is approximately equal to the average of all the time horizons.

Information from the survey indicates therefore that individuals use different discount rates depending on the trade-off proposed between present and future. The pattern that seems to follow the discount rate over time is that of a convex curve with a negative slope.8

Hyperbolic discounting has been criticized because it implies inconsistent intertemporal preferences as individuals change their discount rate when situated in different years, which is not the case with exponential discounting. This being true, these discount rates appear to conform to the stated preferences of individuals subjected to hypothetical choices between present and future.

Another contribution regarding the determination of the social discount rate is called gamma discount and its rationale is as follows (Weitzman, 2001): from 2000 interviews with economists from 48 different countries, and from the values of their responses on the type of discount, Weitzman obtained the distribution of discount rates, which ranged from zero to a maximum of 20 per cent. The distribution is of type gamma with higher frequencies between 3 per cent and 5 per cent, and a long tail on the right with very low frequencies.

Weitzman noted the error of averaging from the distribution of individual rates in order to obtain the social rate of discount. We must average discount factors, not discount rates. Let us see the logic of this proposal with an example. Consider a project that consists exclusively of future benefits of $100 million within 300 years. If the discount rate is 1 per cent the present value of these benefits is equal to $5,053,449. If the discount rate is 10 per cent, the present value is practically zero ($0.00004). In addition to showing the effects of exponential discounting on future benefits, this example illustrates the effect of averaging the interest rates instead of the discount factors.

Suppose that the economy is composed of two individuals, A and B, with discount rates of 1 per cent and 10 per cent respectively, and that the
government uses, as a social discount rate, the average for discounting the benefits of $100 million in 300 years. If we apply the average rate of the two individual discount rates (5.5 per cent), the result is a present value of $10.60, which does not seem reasonable given that individual A values the $100 million as $5 million in the present and individual B gives a value close to zero. The average of these two values is $2.5 million, a figure significantly higher than the $10.60 that results from applying the average rate.

The key is that we cannot average rates but discount factors. If we calculate the implicit discount rate, which is to convert $100 million received within 300 years into $2.5 million in the present, we obtain a discount rate of 1.2 per cent, very close to 1 per cent, the minimum value of the distribution of types (see Table 8.2).

Table 8.2 shows how, in the early years, the outcome of averaging the discount rates does not produce very different results compared with the average of discount factors. However, in a relatively close time horizon of 30 years, using the average of the discount factors implies doubling the benefits that result from applying an average discount rate. It can be seen how the implicit discount rate, which is the average of the present values that both individuals attach to $100 million, decreases with time, approaching the lowest value in the range of discount rates.

**THINGS TO REMEMBER**

- A positive \( NPV \) is a necessary condition for the approval of a project. When projects are mutually exclusive the use of the \( NPV \)
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requires a homogenization of projects. Moreover a positive $NPV$ requires us to examine the optimal timing of the project. Postponing the project may be socially profitable.

- The marginal rate of time preference, the marginal productivity of capital and the interest rate are not equal unless very unlikely circumstances occur. Hence the selection of the social discount rate is difficult and controversial. The practitioner takes the discount rate as a parameter given by the government, and can introduce several plausible values to see how sensitive the results are with respect to changes in the discount rate.

- It is not clear whether exponential discounting is appropriate for dealing with benefits and costs affecting future generations. Many argue that exponential discounting gives too little value to the future and hence it does not reflect individuals’ preferences, but if future generations are expected to be significantly wealthier than we are the case for discounting is stronger. The use of lower rates of discount for environmental impacts or life savings is already being introduced in conventional cost–benefit analysis.

NOTES

1. The discounted sum ($S$) of a unit of net benefit during $T$ years is $S = (1 - (1 + i)^{-T})/i$; for $i = 0.1$ and $T = 25$, $S = 9.07704$; with $T = 50$, $S = 9.91481$.


3. When the decision maker is risk averse, the utility of the expected value is higher than the expected utility. For the analysis of uncertainty in the investment projects and the public sector attitude toward risk, see Chapter 9.


5. See Evans (2007); Pearce and Ulph (1999); European Commission (2008); HM Treasury (2003); and other cost–benefit analysis guidelines.

6. Although it is also argued that, in the case of discounting the benefits from the reduction of deaths and injuries, the marginal rate of time preference does not necessarily coincide with the one used for the discount of other kind of benefits and costs.

7. Which on the other hand may seem reasonable, because by saving $3$ today we will have $117$ million within 300 years if the interest rate is 6 per cent. The problem is that there is no guarantee of maintaining during 300 years the flow of benefits that would, within 300 years, compensate the individuals living at that date.

8. The answers given by respondents to the contingency of saving anonymous lives in the present or in 5, 10, 25, 50 or 100 years (for each respondent only one option was given) allow us to infer a possible implicit discount rate from 16.8 per cent in the lives saved in year 5, to 11.2 per cent in year 10, 7.4 per cent in year 25, 4.8 per cent in year 50 and 3.8 per cent in the year 100.
9. Uncertainty and risk analysis

Nothing is more soothing or more persuasive than the computer screen, with its imposing arrays of numbers, glowing colors, and elegantly structured graphs. As we stare at the passing show, we become so absorbed that we tend to forget that the computer only answers questions; it does not ask them. Whenever we ignore that truth, the computer supports us in our conceptual errors. Those who live only by the numbers may find that the computer has simply replaced the oracles to whom people resorted in ancient times for guidance in risk management and decision-making.

(Peter L. Bernstein, 1996, p. 336)

9.1 INTRODUCTION

The future is uncertain and uncertainty means variability in the outcome associated with a given action. It is impossible to live without risk. Driving to work or purchasing financial assets, for example, are actions associated with the risk of suffering an accident or losing income. Individuals who want to reduce risk in their lives can do so to some extent by insuring themselves and thus reducing the variability of the possible outcomes, and by avoiding the exposure to risk. In both cases individuals pay to reduce the risk to more bearable levels.

An individual whose hobby is mountain climbing can reduce the risk of death by reducing the difficulty of the challenges, or simply by leaving this risky sport. In exchange the individual would have to pay a price in terms of the resulting loss of utility. Similarly buying insurance can reduce the risk of a severe loss of income if the house burns down. The price to pay in this case is the annual insurance premium.

The approach to risk in the private sector is the content of section 9.2. Private investors decide which projects to undertake, taking into consideration the risk involved. A risky project is in principle less attractive unless the higher returns compensate for the likelihood of losing money if the circumstances are unfavourable. Investors require higher rates of return for riskier projects, as compensation for assuming the probability of a loss.

When undertaking a project with a 30-year lifespan whose annual net benefits depend on various contingencies such as rainy or dry weather, or the economy growing at 2 or 3 per cent, part of the project’s success will
inevitably depend on our luck; however the results are not always random. As Bernstein (1996) points out, there is a difference between gambling and those games in which skill influences the outcome. The same principles apply to roulette, dice or slot machines; but something else is required to explain the results in poker or betting on horses. It could be that the newcomer wins a round, but when the game is repeated many times, eventually the professional wins.

When an entrepreneur assesses the purchase of a financial asset, which consists of an initial payment of $1000 in year zero for $1100 in exchange in year one, he is comparing a certain present amount with some other certain amount in the future. The asset has a safe yield of 10 per cent.

Suppose that the entrepreneur has also the option to invest $1000 in a business from which, depending on the economic cycle, he would gain $1600 if the economy is ‘doing well’ and recovering the $1000 if the economy ‘goes wrong’, with a probability of 0.5 for both outcomes.

Given the two contingencies and associated probabilities, the expected value equals $1300 (i.e. the return is $0 or $600 with equal probability). However, if the investor is risk averse, he does not consider the expected value as a safe amount. It may well be that the individual decides to buy the asset where a $100 return can be obtained with certainty, as opposed to the asset that offers an expected yield of $300. The explanation is: in the risky asset he can obtain $0 or $600 with equal probability, but never $300, in contrast with the certain $100 in the first option. If he is indifferent between both assets, his certainty equivalent of an expected profit of $300 is $100 or less.

The idea of a certainty equivalent lower than the expected value is similar to the discount of future risky benefits at a discount rate higher than the interest rate. The only reason for a risk averse investor to accept a risky business is that the expected return, once discounted with a higher rate (it includes a risk premium), is positive.

A key question we address in section 9.3 is whether the public sector should act in the same way as the private sector by calculating expected values with a higher discount that includes a risk premium, or whether it should behave differently.

Complete certainty does not exist, therefore investment projects or public policies that have lasting effects require predictions of their impacts and estimations of their magnitude. The uncertainty associated with the benefits and costs of a project indicates that the results rest on a range of values and their associated probabilities, rather than on deterministic values. Sections 9.4 and 9.5 contain the fundamentals of risk analysis, a tool that, instead of using deterministic values, introduces ranges of feasible values of key variables and their probabilities of occurrence. Risk analysis provides useful information to enhance decision making.
9.2 RISK IN A PRIVATE PROJECT

A common approach to dealing with risk is to distinguish between risk and uncertainty. The decision under risk would be one in which the individual knows the true probabilities assigned to individual events; for example the probability of obtaining two when throwing a dice is one in six (if the game is repeated a sufficiently high number of times, we obtain two approximately 16.67 per cent of the time). On the contrary, under uncertainty, we do not have an objective set of probabilities associated with the different possible cases, for example when predicting the next technological breakthrough in energy.

From the perspective of project evaluation this distinction is not very useful. Decision makers live in a world of uncertainty where their degree of belief concerning the different possible relevant cases under consideration can be subject to hard or soft probabilities (Hirshleifer and Riley, 1992). This subjective probability concept applies equally to tossing a fair coin or a coin that could also have two tails or two heads. In both cases one has to work with the same probability function – one half for each state (tail or head) – though in the case of a previously tested coin we are working with hard (objective) probabilities.

The relevance of this distinction for project evaluation is linked to two types of actions the individual faces under uncertainty: terminal action and informational action. In a situation of terminal action it does not matter whether the individual knows a priori the number of heads on the coin to be tossed. On the contrary, in a context of informational action, the investor could be interested in spending money to obtain some information on the type of coin he is going to toss.

The decision maker has to move between these actions in project appraisal depending on the circumstances. He has to evaluate the project with subjective probabilities associated with different possible states but sometimes can improve confidence in the future possible outcomes by investing money and effort to acquire additional information on the true probabilities associated with the different states of nature (for example designing contracts to increase the probability of selecting an efficient constructor and therefore reducing the project costs).

The perceived risk is dependent on the analysis of the so-called a priori information, the expert opinion and the judgement of the analyst who interprets all the information at hand. In a public tender for granting the concession of a road there will probably be different traffic forecasts; one cannot assert ex ante that one is better than another because, from a common fact, different bidders assign different probabilities to the various contingencies that can be considered to determine the volume of traffic.2
An individual deciding whether to invest the amount $I$ in a project needs to estimate its net present value, to check whether it is greater than or equal to zero and so to satisfy the following condition:

$$\sum_{t=1}^{T} \frac{(p_t x_t - C_t)}{(1 + i)^t} \geq I_0,$$

(9.1)

where $I_0$ is the investment cost (realized in year zero), $p_t$ the price, $C_t$ the annual cost and $x_t$ the quantity during the $T$ years of the project, and $i$ denotes the discount rate.

Expression (9.1) represents an investment project in which there are no other fixed costs than the investment cost in year zero. We must estimate the costs of initial investment (oil drilling, launching a new product, building a dam, etc.) and predict the benefits and costs over the $T$-year horizon of the project. If $T$ is sufficiently high, it can be virtually assured that the values finally taken by the main variables, such as input and output prices, will differ from those initially predicted.

Let us consider the case of a private project involving the construction of a complex of 150 apartments for renting in a tourist resort. The cost of construction is budgeted at $13,000 and the annual net profit over the lifetime of the project ($T = 15$) depends on the number of rooms occupied ($x$), the rent per apartment and year ($p$) and annual maintenance and operation costs ($C$). We are facing a risky investment. The revenues and costs of construction and operation of the apartments can vary for several reasons, including changes in the domestic and the global economy (effect on costs), changes in the economy of countries of origin of tourists (effect on demand) and variation in the prices of competitors (effect on demand).

To incorporate into the analysis the uncertainty that has to face an entrepreneur who is evaluating whether to invest in the project, assume that the annual demand only takes two values: high, which means a maximum daily occupation ($x = 150$), and low ($x = 125$). Both states are equally likely, and we initially assume that once a value is observed, high or low, in the first year, it will remain for the rest of the coming years. The annual operating cost is $400 regardless of occupancy rates, and the rent per apartment and year is $10.

The expected net present value of the project can be expressed as:

$$NPV = -13,000 + \frac{(0.5 \times 1100 + 0.5 \times 850)}{1 + i} + \ldots + \frac{(0.5 \times 1100 + 0.5 \times 850)}{(1 + i)^{15}}.$$

(9.2)
Assuming for simplicity that the real interest rate is zero, the expected \( NPV \) is equal to:

\[
NPV = -13,000 + (975 \times 15) = 1625.
\]  

To understand how uncertainty affects the project let us think about what a profit of $1625 means. This value is not an actual return; it is an expected return, expected in the sense of mathematical expectation; that is, the value to which the benefit tends if this project is repeated to infinity. The real return that the investor receives from the project is either a $3500 gain if the demand is high or a $250 loss if the demand is low. These are the real values the investor will receive: $3500 or –$250.

If it were possible to have full insurance at zero cost, the fair premium equals $1875 (the difference between the result with high demand and the expected value). This is a fully comprehensive insurance that guarantees the expected value of $1625 in any of the contingencies: high or low demand.

With this comprehensive insurance, the variability of the results disappears, and a risk averse investor will buy the insurance to guarantee the expected value. It is worth emphasizing that, once insured, he will not incur losses, but neither will he get the maximum benefits. A risk loving entrepreneur would reject the offer of the insurance company to have the opportunity to win $3500.

In Figure 9.1 we can see the profits with low demand (–$250), the profits with high demand ($3500), the expected value ($1625) and two new values ($1250 and $200) explained below.

In the initial situation, without insurance, let us suppose now that a tour operator offers the investor the following contract: full occupancy rent of the apartments is ensured during the 15 years in exchange for a payment below the ‘free market rent’. The guaranteed price per apartment will be $9 for the 150 apartments regardless of how many are occupied (the

\[\begin{array}{c}
\text{\$250} & \text{\$1425} & \text{\$1625} & \text{\$3500} \\
\text{\$1875} & \text{\$2250} & \text{\$3300}
\end{array}\]

Figure 9.1  Profits under uncertainty
entrepreneur is not allowed to rent the unoccupied ones). The demand may still be high or low, but now, in any event, the entrepreneur receives a fixed payment of $1250 located in Figure 9.1 on the left of the expected value:

$$NPV = -13,000 + (950 \times 15) = 1250.$$  

(9.4)

Would the entrepreneur accept this offer? We cannot answer this question without knowing the attitude toward risk of the entrepreneur; however a simple observation of reality shows that many individuals accept similar offers every day. They would be willing to accept a smaller but certain amount (e.g. $1250) by giving up a higher expected value ($1625). They are risk averse and there exists a safe value (certainty equivalent) lower than the expected value for which the entrepreneurs are indifferent between accepting the safe value or playing the game consisting of a potential gain of $3500 and a potential loss of $250. The offer of the tour operator is similar to fully comprehensive insurance where the entrepreneur pays a $2250 premium in exchange for having a safe profit of $1250.

Suppose now that the certainty equivalent of the entrepreneur is $200 as represented in Figure 9.1; that is, he is indifferent between receiving $200 of safe benefits against the possibility of winning $3500 or losing $250 with equal probability. This means that, although the expected value of the investment is $1625, the variability associated with this expected value (profit of $3500 or loss of $250) represents a cost to the individual of $1425, which makes him indifferent between the safe profit of $200 and the expected value of $1625.

This is equivalent to saying that the entrepreneur is indifferent between paying a premium of $3300 in order to ensure the maximum profit of $3500, or investing without insurance. As the tour operator is offering a better contract – asking for less ($2250) than the entrepreneur is willing to pay ($3300) – the entrepreneur will accept the offer.

From the arguments above, we conclude that a risk averse private investor does not use the expected value of annual net profits for the calculation of the $NPV$, but the certainty equivalent corresponding to the expected value of each year, certainty equivalents that will be smaller than the annual expected values, with a difference increasing with the degree of the investor’s risk aversion.

Instead of assigning certainty equivalents corresponding to the expected values of annual net benefits, the treatment of risk in private projects is sometimes based on raising the interest rate that is used to discount the expected annual net benefits; that is, introducing a risk premium, which is added to the interest rate.
While increasing the interest rate reduces the profitability of the project, this approach is not the same as using certainty equivalents. Adding a constant risk premium to the interest rate implicitly assumes that uncertainty grows exponentially with time and this may well not be the case; sometimes the risk is higher during the first years of the project, or may not reflect the risk attitude of individuals.\textsuperscript{3} It is therefore better to work with the certainty equivalents of benefits and costs, and discount the flows with the risk free discount rate.

9.3 RISK IN THE PUBLIC SECTOR

There are economic reasons that support a different treatment of risk when projects are evaluated in the public sector. We have seen that, for a private investor who is risk averse, there is a safe level of profitability (the certainty equivalent) lower than the expected value, which leaves the investor indifferent between the risky project and a risk free return.

For the private entrepreneur it makes sense to discount the profits at a more demanding rate, because the risk is a real cost for a risk averse investor. In Figure 9.1 the expected profitability of the project is $1625, but the certainty equivalent is $200. The difference ($1425) is the subjective cost of risk. This cost is a real cost to the society, since the benefits and costs are measured by what individuals are willing to pay for goods and services and what the individuals are willing to pay to avoid the risk. In Figure 9.1 the entrepreneur accepts the offer because the tour operator asks for a lower price than the one the entrepreneur is willing to pay for the risk transfer to a third party.

Should the public sector maximize the \(NPV\) of the net expected social benefits of investment projects and public policies? Or should it maximize the \(NPV\) of the net social benefits adjusted for risk?

The theoretical foundation to discard the risk approach in public sector projects is supported by the theorem of Arrow and Lind (1970). Positions prior to the Arrow–Lind theorem about whether the public sector should introduce risk premiums into the discount rate, as the private sector does, are the following:

- The public sector should use the same risk adjusted discount rate as the private sector because, if the public sector uses a risk free discount rate while the private sector introduces a risk premium, there will be a misallocation of resources by overinvestment in the public sector.
- The public sector should ignore uncertainty and act as if it were indifferent to risk. Project evaluation should be performed with the expected values and a risk free discount rate. Given that many similar and independent projects are carried out, the results will tend to the expected value. This can be interpreted as if the public sector is insuring itself by paying for the losses in adverse situations with the gains in favourable ones.

In contrast with the second argument, Arrow and Lind’s theorem is not based on the diversification of risk (by investing in a large number of projects the return will tend to the expected value) but it is based on the idea of risk spreading, as the difference between the expected value and the certainty equivalent tends to zero when the benefits (or losses) of the project are divided among a large number of participants (taxpayers), and then the cost of risk tends to zero.

It is true that, unless the individuals are risk neutrals or risk lovers, the expected net present value overestimates what those individuals are willing to pay. The social cost of risk will depend on the preferences of the individuals who enjoy the benefits and pay the costs, and the relative importance of these costs and benefits with respect to their wealth.

What Arrow and Lind’s theorem shows for the economic evaluation of projects is the following: suppose a world in which the government charges the beneficiaries their willingness to pay and the final net benefits, either positive or negative, are equally distributed among the taxpayers. As long as the impact of the project on their income is very small, the social cost of risk tends to zero and the expected value is close to the true benefit of the project when the number of people affected is large enough. The argument also applies to the case of a major project, as long as it involves a negligible change in the taxpayers’ income.

In the real world, costs and benefits are not always distributed in that way. Too often the government assumes the investment costs of the project and the individuals receive the benefits (and some other costs such as environmental impacts, accidents, etc.) whose magnitude can be equivalent to a significant proportion of their income. In many projects there will be costs paid by taxpayers, and significant costs and benefits paid/received by the individuals.

According to Arrow and Lind’s theorem, in the calculus of the NPV of the project, the costs and benefits of the public sector are reckoned in terms of their expected values and using a discount rate without the risk premium; on the contrary, if the costs and benefits are borne by the private sector, some adjustments have to be made reflecting their preferences, which means using the certainty equivalents instead of the expected values.
An additional difficulty emerges because of the aggregated nature of the data used in the evaluation of projects. Given that costs and benefits are frequently aggregated they may hide different effects on different groups of individuals. Moreover, aggregated social benefits may include benefits for some and costs for others. In this case, the adjustment to account for risk is more complicated. If, as in many public projects, the investment and operating costs are borne by taxpayers and the benefits (and other costs) are borne by groups of individuals, the investment and operating costs should be discounted with the risk-free discount rate and the benefits (and other costs) to individuals should be adjusted downward (upward) according to the uncertainty with which they are associated.

Johansson and Kriström (2009) warn of the use of certainty equivalents when there are other costs that could be easily ignored. Consider the following proposed project. A hydropower plant was asked to reduce its production by 3.7 gigawatt hours (GWh) annually. The benefits of this project are environmental and recreational. A web-based contingent valuation study was undertaken among people living along the river. A referendum-style willingness-to-pay question was asked and in total the residents were willing to pay around $2.6m, expressed as a present value.

The Nordic (Nord Pool) spot market for electricity is illustrated in Figure 9.2. There are different production technologies, each assumed to have constant marginal costs. Hydropower and wind power have very low marginal costs. Nuclear power is intermediate and fossil-fired plants have the highest marginal costs. The market price is $50,000 per GWh. In reality there are more alternatives available and demand fluctuates over time, implying that the figure provides a highly simplified picture of the market.

The owner of the hydropower plant seems willing to accept a once-and-for-all payment; that is, a certainty equivalent, of $2.6m in exchange for reducing annual electricity production by 3.7 GWh. This is a kind of certainty equivalent since the alternative is associated with price uncertainty and the firm uses a much higher discount rate (>7 per cent) than society (3 per cent in this case). So it seems as if a deal is possible. The local residents – or the taxpayers – could pay for the loss of profits. And certainly, nobody can prevent two parties making such a deal. Still, the question is if the project is socially profitable.

The social cost is the cost of replacing 3.7 GWh annually. In terms of Figure 9.2 the ‘supply ladder’ shifts to the left by 3.7 GWh. The annual cost of replacing almost cost-free hydropower by fossil-based electricity with a marginal cost of $50,000 per GWh is $50,000 × 3.7 = $185,000. If the social discount rate is 3 per cent, then the present value cost is around $6m. According to this analysis, the proposed project is highly unprofitable for society.

From the point of view of society we have to crowd-out consumption
worth $185,000 annually in order to release resources that can produce the
replacement power. The present value of this cost is $6m if the discount
rate is 3 per cent. So in addition to the $2.6m that is supposed to be paid
for the project, an additional $3.4m (6 − 2.6) of consumption must be
sacrificed. The resources needed to produce the replacement power have
an alternative use. In addition we replace ‘clean’ hydropower with ‘dirty’
fossil-based electricity so there is an externality involved too. In any case,
even if one involved party (taxpayers or local residents) can pay another
party (the firm), there might be other effects that should be accounted for
in a cost–benefit analysis. Financial or distributional effects might hide the
true benefits and costs of a project.

The lesson from this project is that we need to include all the costs and
the affected agents in cost–benefit analysis. The conclusion in Johansson
and Kriström (2009) is equivalent to saying that we respect the private
preferences (certainty equivalents of the hydropower firm and the resi-
dents) and then include any other costs, like the cost of the replacement
electricity at a higher social cost. So, we maintain the individual prefer-
ences (using their willingness to pay/willingness to accept that include the
cost of risk), and then use the risk free social discount rate.

9.4 RISK ANALYSIS

Having analysed the differences in the treatment of risk between the private
and the public sector, let us consider the available options to acquire more
information on the risk associated with the project under evaluation.
In the economic evaluation of projects, the use of a deterministic analysis is not uncommon, choosing unique values for quantities and prices, values that come from the best available information and are treated as if they were to be realized in the future.

However reality does not support the calculation of the \(NPV\) under the assumption that variables are deterministic. The uncertainty surrounding demand and costs recommends the inclusion of random variables that allow us to obtain a range of net present values and their corresponding associated probabilities. Before we describe risk analysis let us briefly examine the rationale of sensitivity analysis.

**Sensitivity Analysis**

Sensitivity analysis consists of changing the value of one variable and checking how this affects the outcome of the project. When several variables are modified simultaneously we are using scenarios.

Returning to the case analysed in section 9.2, 150 apartments are built for which full occupation is expected. For simplicity no externalities are added, shadow prices are not considered and the discount rate is set to zero. The social benefit of the project is $3500; so, if the decision is to approve or reject, the apartment complex project is approved.

Let us see the sensitivity of the \(NPV\) of this project to alternative changes of \(x\) and \(C\). For example when the demand is 125 the profits are \(-$250\), and when the costs equal $600 the profits are $500. Therefore the analyst knows that when the occupancy rate falls and only 125 flats are occupied, the outcome equals losses of $250. The selection of 125 is ad hoc based on the a priori belief that this value is the worst situation the investor thinks possible.

An option within the sensitivity analysis is to calculate the threshold values of the relevant variables. This check of the sensitivity of the results is performed by changing the value of the selected variable until the \(NPV\) is zero, and it is often presented in relative terms, as the percentage change in the target variable that makes the benefits of the project equal to zero. For example, in the case of the construction costs, the maximum increase before the \(NPV\) becomes negative is 26.9 per cent, the reduction in price is 15.6 per cent and the increase in annual costs 58.3 per cent.

The rationale of scenarios is not substantially different to sensitivity analysis. Instead of modifying a single variable and holding the others fixed, in scenario analysis we look at the combined effect of changes in some selected variables, changes corresponding to different possible scenarios. For example we can change three variables altogether (construction costs, annual operating costs and demand). The scenarios are
called, for example, ‘optimistic’ (high demand and low cost), ‘expected’ (average values for costs and demand) and ‘pessimistic’ (high costs and low demand).

The sensitivity analysis and the use of scenarios have the advantage of revealing the degree of robustness of the results obtained by the change in the value of a variable or a set of them, and comparing the results with those obtained in the deterministic analysis. In our example, when demand falls 16 per cent from the initial value of 150, the project loses money. This simple sensitivity analysis indicates that, before undertaking the project, we should put greater effort into obtaining a more reliable forecast of demand, or even seek a contract that guarantees revenue in some way.

Once the usefulness of sensitivity analysis and scenarios is acknowledged, we must note their limitations because of the use of unique values instead of a range of values and the likelihood of the $NPV$ being positive or negative given these other values within the feasible range. The use of single values does not imply an objective system. By choosing values of individual variables, the interrelationship between them can be neglected. The randomness of many of the events that affect the project will result in a joint realization of the values of variables that does not need to conform to a rigid choice of scenarios.

**Risk Analysis**

The alternative cannot be based on the addition of more variables to the sensitivity analysis as the number of possible combinations multiplies, providing too much information of little practical use to the decision taker. The alternative to the sensitivity analysis and scenarios consists of analysing the impact of the variables on the $NPV$ of the project in a more systematic way. The alternative approach is called risk analysis. In the following we describe risk analysis using the previous example.

In sensitivity analysis the demand could take the value of 150 in the best case and 125 in the worst case. An alternative to the previous approach is to work with probability distributions, allowing all possible values between 125 and 150. This introduces more realism into the assessment. The main point is that in the sensitivity analysis we fix the demand value to 150 or 125 but we do not use our beliefs on the likelihood of any of these values or any other possible value in the range. If, given the available information, it is estimated for example that the most likely value is 140, the minimum is 125 and the maximum 150, we could use a triangular distribution function like the one in Figure 9.3.

Risk analysis seeks to make the most of the available information. If some information is available on the a priori reasonable range over which
the relevant variables are expected to fall, it is preferable to use all the values, weighted by the probability of occurrence. Any software for risk analysis will carry out in a short time a very high number of iterations, each one computing a different value for the \( \text{NPV} \). This is because the software will randomly select a number of apartments occupied according to the information provided by the triangular distribution in Figure 9.3, or any other, previously selected. The most frequently chosen values will be 140 and those close to this value.

Instead of computing a single \( \text{NPV} \) and then performing sensitivity analysis with two or three values of demand to see how the profit changes, now we can have a very high number of different \( \text{NPV} \)s, which were obtained with random demand values within the range 125-150 and their assigned probabilities of occurrence in the selected probability distribution function. The risk analysis is based on the following four stages.4

1. Modelling the project

This stage is actually common to any financial or cost–benefit analysis, either deterministic or stochastic (incorporating uncertainty). It consists of constructing a model that captures the relationship between costs and benefits over time in order to predict the \( \text{NPV} \) of the project according to the values taken by the variables.

In the case of the project of the apartment complex the model is simple. It has four variables – construction costs, operating costs, the number of apartments occupied and price – that may take different values over the 15 years of the project life.

In general the decision on the variables of the project depends on its characteristics. Suppose a contract was signed with the construction firm
fixing the price and the construction deadline. The contract includes a clause providing for an automatic review of the price if during the years of construction the labour costs change. If the labour costs are expected to vary and they are a significant part of total costs, the price of labour should be a random variable of the model.

It could also happen that the construction costs are sensitive to certain features of the terrain, impossible to determine accurately a priori, so it would be advisable to use two variables: the cost of materials used and labour costs.

2. Selection of risk variables
Of all the variables that determine the profitability of the project, one should choose only those that, besides being likely to change, if they do, the results of the project are significantly modified. Therefore the variables that meet one of the following two conditions could be excluded: (i) they have a high impact if they change, but it is unlikely that they will change, or (ii) they are likely to change, but if they do, their impact is irrelevant.

The reason to reduce as much as possible the number of variables included in the risk analysis is that the more variables are included, the more difficult it is to establish correlations between them and the more likely it is that the results will be inconsistent when random simulations are generated. Furthermore, by reducing the number of variables, we can focus our effort on the behaviour and the interactions of the selected variables.

3. Probability distributions for the variables
Under the presence of uncertainty it is difficult to determine the value of the variables of the model. Knowing the exact occupancy rate during the next 15 years is virtually impossible; however we may be able to determine the expected range for the percentage of occupied apartments in the 15 years. If, given the situation and characteristics of the complex, with information based on past experience and expert opinion, we predict that the occupancy rate will be in the range between 70 per cent and 90 per cent, for example, we are in a better position with respect to the situation in which we use only two values.

Once we have the minimum and maximum values, we must decide which probability distribution is the most appropriate. The choice of the range and type of probability function is indeed a prediction of the future that is based on data from the past and also on our subjective view of what we think the future will be like. 5

Suppose that in the case of the apartments, the statistical information on occupation rates in recent decades and the information gathered through interviews with experts do not provide any reasonable belief that
one particular value within the feasible range is more likely than the rest. We only know the minimum (125) and the maximum (150). In this case the uniform probability distribution (Figure 9.4) is the one that reflects our a priori beliefs.

Given this probability distribution, it does not seem reasonable to perform the risk analysis with two or three values, as in the sensitivity analysis or with scenarios. The discrete uniform probability distribution of Figure 9.4 shows 26 equally likely values. Given this expected demand pattern, the available information on the extreme values helped to build the probability function that is used to perform simulations. In many cases, given the nature of the project, it may happen that one has to choose a probability distribution knowing only the most likely value and having a rough idea of the changes on both sides of the mean.

For example it may happen that we expect the demand quantity for a service to be around a mean value of 100 and a standard deviation of 5, with the probability distribution approximately symmetrical. In this case a normal distribution is appropriate. In general the symmetric distributions should be used when the value of the variable ultimately depends on opposing forces of a similar weight, while the asymmetric distributions reflect situations where there is rigidity on one side of the distribution. If it is expected that the price of land is located in a range in which higher values are more likely, we should use an asymmetric distribution.

In many cases a uniform distribution (i.e. an equal probability assigned to each value) as represented in Figure 9.4 may be appropriate if there is no evidence that supports assigning a greater weight to any of the values within the range formed by the minimum and maximum. The uniform
distribution is therefore the baseline, the one compatible with the lowest level of information.

4. Correlated variables
Risk analysis is based on a computer program in which the specified model to calculate the \(NPV\) is run many times, taking in each iteration the fixed value of deterministic variables and randomly drawing a value for the risk variables according to the selected probability distribution. At the end of the process, the program yields a range of \(NPV\)s with their respective probabilities.

In the computation process, the program draws a value for each risk variable regardless of the value chosen for the others, a procedure that can lead to inconsistent results since, if some variables are correlated, this relationship should be included in the program if we want to avoid inconsistent outcomes. If airport delay depends on the ratio ‘flights per hour/airport capacity’, it makes no sense that the program can choose a high waiting time value and simultaneously a low value for the ratio. The procedure to prevent the program generating inconsistent results is to create a correlation matrix in which the relationship between one variable and the others is reflected.

9.5 INTERPRETING THE RESULTS OF RISK ANALYSIS

The model of our example was as simple as:

\[
NPV = -I_0 + \sum_{t=1}^{15} (p_t x_t - C_t). \tag{9.5}
\]

Risk analysis uses the previously selected deterministic variables. In our example \(I\) is always equal to 13,000, \(p\) is equal to 10 and \(C\) is equal to 400. These three variables are not subject to variability and any time we run the program an \(NPV\) value is generated with these predetermined values. On the contrary, \(x\) is the risk variable. The probability distribution for the risk variable is represented in Figure 9.4, a uniform distribution with a minimum at 125 and a maximum at 150. We assign the same probability to any of the 26 possible values that can take the variable demand in a given year.

Once the software has generated a high enough number of iterations, the results can be displayed as a probability distribution function, which represents the probability distribution associated with the \(NPV\) of the
project, and it will allow us to calculate the probability of the $NPV$ being above or below a certain value or within a range of values.

The $NPV$ of the project is not now a single number that receives a greater or lower significance depending on the risk aversion of the decision maker. With the risk analysis we have a probability distribution of the $NPV$ of the project that contributes to a more informed decision. Obviously the risk of the project is exactly the same with the simple analysis of expected values, the sensitivity test, the use of scenarios or Monte Carlo analysis, but the risk of making a wrong decision diminishes after a well conducted risk analysis.

Recall that in our case of the investment in apartments an evaluation of the project based on the expected value of the benefits and an annual demand given by either 125 or 150 with equal probability results in an expected profit of $1625 (see Figure 9.1). If the demand takes the highest value the benefits are equal to $3500, while if it takes the lowest value the project results in losses of $250.

Suppose alternatively that the value the demand takes in a year does not determine the demand in the following years. Under this assumption we have to make the draw from the random variable $x$ for each of the 15 years of the project. The $NPV$ obtained in a given iteration will be the result of drawing 15 values of $x$ within the 26 possible ones for each of the 15 years. This process is routinely repeated by the computer thousands, or hundreds of thousands, of times.

We must emphasize that the risk analysis does not only provide $NPV$ values but also their probability of occurrence. If I had to invest 50 per cent of my annual salary in a business with only one potential negative outcome, I would be interested in knowing whether the probability of occurrence of the adverse outcome is similar to the one of obtaining a head in tossing a coin, or winning the first prize in the national lottery.

Figure 9.5 shows the probability distribution of the benefits corresponding to 100,000 iterations of the model; it is the picture of 100,000 $NPV$ values obtained by randomly drawing 100,000 values for the demand for each of the 15 years. The mean $NPV$ is $1625, which coincides with the one obtained with the expected value of demand (see Figure 9.1). This does not add any value to what we already know. Nevertheless the distribution of the net present values provides new valuable information. If our assumption on the demand’s behaviour is correct, we know now that a negative $NPV$ is unlikely. Why? Because negative $NPV$ values require that low demand values are drawn for many years, which is certainly unlikely.

The probability distribution of profits represented in Figure 9.5 shows that a negative result is almost impossible, since the minimum value is $420 and the maximum $2880, with an expected value of $1625 and a
standard deviation of $291. However we know that there are demand values that generate losses. When the demand is 125, the loss is $250, and as Figure 9.4 shows the value 125 is as likely as any other. The paradox is solved when one recalls that for a negative profit of $250 the demand must take the lowest value (125) during the 15 years.6

What the risk analysis shows, after repeating the calculation of the NPV a sufficiently high number of times, is the impossibility of such a negative result. The variable demand takes the value 125 in a given year 3.85 per cent of the time, but the number of drawings that are made is 15, and in these 15 subsequent and independent extractions the value 125 must be realized. The trend is toward the mean, and this is what Figure 9.5 represents.

The cumulative probability distribution represented in Figure 9.6 is another way to show that this is not a risky project because there is no case in which the returns are less than $420. Figure 9.7 shows the curve fitted to the histogram of Figure 9.5. The adjusted density shows that, for example, 99 per cent of the potential net present values are in the range of positive profits (890; 2360), with virtually zero probability of having negative results.

Should we rule out the possibility of losses in this project? The answer is yes if the model we have used to represent the investment in apartments is appropriate and reflects the behaviour of the variables. Risk analysis software works with the inputs it receives from the user, but cannot discriminate between a model of realistic behaviour and another based on inadequate assumptions.

The key assumption regarding the risk variable in our example is that
the demand is random each year, regardless of what happened in the previous year, and also that the number of occupied apartments will never be less than 125 out of the 150 to be constructed, any of them being equally probable, including the minimum and maximum.

To illustrate the importance of the assumptions of the model, suppose alternatively, as we did at the beginning of the chapter, that the demand is random only in the first year within the same limits of 125 and 150, and once a value of \( x \) has been drawn for the first year, it remains constant.

\[
\begin{align*}
&\begin{array}{c}
0.0016 \\
0.5 \\
1 
\end{array} \\
&\begin{array}{c}
420 \\
1625 \\
2880 
\end{array}
\end{align*}
\]

**Figure 9.6** Cumulative probability distribution of the NPV (one uniform probability distribution of demand for each year of the project)

**Figure 9.7** Probability density function of the NPV (one uniform probability distribution of demand for each year of the project)
during the following years. If this is the assumption that represents the real world we are trying to model, the results change dramatically. Although the mean is the same ($1625), the minimum and maximum NPV values change ($−250 and $3500) with a standard deviation of $1125 ($291 in the previous case).

The drastic change in the results is represented in Figures 9.8 and 9.9, which show two fundamental differences with respect to the previous outcome. First, the project presents the possibility of higher returns (an NPV of $3500 is as likely as the mean) and, second, it increases the
variability of the results (including the possibility of losses: −$250 is as likely as any other within the range).

As expected, the same data lead to different results depending on the assumptions about the behaviour of the demand. From the previous analysis we can appreciate the importance of the effort to develop a consistent model that reflects the real behaviour of the variables as closely as possible, since the consequences of incorrectly modelling such behaviour might have a decisive impact on the evaluation results. This modelling allows the decision taker to work with possible results and their respective probabilities but its relevance depends on how well the model represents the real world.

Sometimes this information is enough to take a decision on the desirability of a project; in other cases nothing can be concluded with a single decision rule, especially when comparing projects. We will now see several possibilities using the results of risk analysis, useful when the decision is either ‘to approve or reject’ or ‘to select between two mutually exclusive projects’.

**Decision Criteria Under Uncertainty**

**Accept or reject a project**

If the decision is to accept or reject a project and the probability distribution of the \( NPV \) does not show any positive value, the criterion is the same as in the case of an evaluation with deterministic variables in which the \( NPV \) is negative: reject the project.

When all the values of the probability distribution of the \( NPV \) are positive, the criterion is, in principle, to accept the project. The ultimate acceptance of the project requires the financial result also to be positive, because if the financial \( NPV \) is negative, or if there is a high probability of negative results, the decision will depend on the budgetary constraint.

When the expected value of the social \( NPV \) is positive and the financial result is acceptable, the project should be approved if the decision maker is risk neutral. However, unlike the situation with deterministic values, we know now the probability of the occurrence of negative results (10 per cent in Figure 9.10) and therefore we have additional information to make a decision that will also depend on the existence of budget constraints and the attitude towards risk aversion of those who take the decision. Examining other pricing policies, capacity size, level of service, and so on may be a way to improve the balance between the project’s social and financial profitability, provided that the foreseeable worsening of the social \( NPV \) does not go beyond acceptable levels.
Choice between projects

The basic criterion for choosing between two alternative projects, A and B, is to select the project whose probability density function shows a clear superiority over the other. In Figure 9.11 the corresponding probability distributions do not intersect and one project is clearly superior to the other. It must be remembered that even in this favourable situation it is necessary to check, in the case of budget constraints, that the financial result is positive (or has a satisfactorily high probability of being positive).

It may occur that the cumulative density distributions of the two projects do not intersect but the density functions do, as in Figure 9.12. In this case the choice may heavily depend on the financial results. In Figure 9.12 project B is preferable to project A because it has a higher expected value and furthermore there are no negative values in the probability density function; however, if the financial results matter, it may be that A is preferable to B if A has a positive financial result and B a negative one.

Let us see the case of two projects whose cumulative density functions intersect. The decision on which project is selected depends, as in the
Introduction to cost–benefit analysis

previous cases, on the social $NPV$, the financial results and the level of risk.

A first criterion for choosing between the two projects represented in Figure 9.13 is to check which project has a higher expected social $NPV$. As Figure 9.13 shows, project $B$ has a higher expected $NPV$ than project $A$. If the financial $NPV$ is positive in both projects, project $B$ is preferred unless the wider range and cumulative probabilities of negative values in project $B$ and the risk aversion of the public agency taking the decision make project $A$ more attractive (a smaller but less risky expected $NPV$).

When the financial $NPV$ of project $A$ is positive and that of project $B$ is negative, it would be preferable to choose $A$ in the presence of a budget constraint. If in addition the risk associated with project $A$ is smaller, as in Figure 9.13, it is more in favour of project $A$.

When the financial $NPV$ is negative for both projects and there are budget constraints, the best option is to look at how the social $NPV$ is affected by changes in the price level, in the capacity design, in the quality of service, and so on. There is a trade-off between a reduction in the social $NPV$ and the improvement in the financial performance.
THINGS TO REMEMBER

- Uncertainty is common to any project, so the economic evaluation of projects has to deal with the variability of results. For risk averse individuals the expected value ignores the cost of risk. People are willing to accept a lower amount than the expected value if by doing so they avoid the variability of the results. Hence the valuation of benefits and costs for specific groups of affected individuals needs an adjustment to account for the cost of risk.

- According to the Arrow–Lind theorem, the public sector should decide with regard to the expected values. The rationale of this position is based on the idea of risk spreading, as the difference between the expected value and the certainty equivalent tends to zero when the results of a project are divided among a large number of participants (taxpayers), so that the cost of risk tends to zero. Nevertheless, in the case of costs borne by specific groups of individuals, the certain equivalent is still the right approach.

- Although the public sector can be considered as risk neutral and bases its decisions on expected values, the information obtained through risk analysis is helpful. Information on the probability of good and bad results gives the decision taker a more complete picture of the project’s expected performance and its consequences.

- A decision of the type accept–reject or a selection between projects can rarely be exclusively based on the social $NPV$. The financial $NPV$ may be useful for making the final decision. Usually we have to face a trade-off consisting of a lower social $NPV$ in exchange for financial viability.

- The best software available for risk analysis does not remove the need to have a good model that represents the case we want to evaluate.

NOTES

1. The certainty equivalent of a gamble is the amount of money that produces the same utility as the expected utility of playing the game. The certainty equivalent leaves the individual indifferent between playing and not playing. Let us consider a game that consists of tossing a coin, winning a million if it is heads and zero if it is tails. The certainty equivalent is the minimum amount of money we have to offer to leave the individual indifferent between accepting this amount and not playing, or rejecting it and playing (see Chapter 11).

2. When several competitors in a public tender have a different view on the auction that has a ‘common value’ (e.g. the stock of crude oil in an oil reservoir), it is said that the winner may suffer the ‘winner’s curse’ and be bankrupt later. The explanation is that if the value
of the object is unique and common to all (unlike a piece of art, for example), the bidder making an offer above the others may have interpreted the information in an optimistic way and overstated the value of the auctioned object.

3. One monetary unit in year one, using a 5 per cent discount rate, has a value of 0.952 in year zero, in years 10 and 15 the values are 0.614 and 0.481 in year zero, respectively. By adding a 3 per cent risk premium the present values are respectively (between brackets, the reduction of the value with respect to the situation without risk): 0.926 (−2.7%); 0.463 (−24.6%); 0.315 (−34.5%).

4. The description of the required stages to account for risk using Monte Carlo simulations is based on Savvides (1994).

5. Bernstein (1996) emphasizes that the story told in his book is marked all the way through by a persistent tension between those who assert that the best decisions are based on quantifications and numbers, determined by the patterns of the past, and those who base their decisions on more subjective degrees of belief about the uncertain future, and points out this is a controversy that has never been resolved.

6. The probability of this event is \((0.0385)^{15} = 6.05 \times 10^{-22}\).
10. Applications

. . . I think we must take it for granted that our estimates of future costs and benefits (particularly the latter) are inevitably subject to a wide margin of error, in the face of which it makes little sense to focus on subtleties aimed at discriminating accurately between investments that might have an expected yield of 10.5 per cent and those that would yield only 10 per cent per annum. As the first order of business we want to be able to distinguish the 10 per cent investments from those yielding 5 or 15 per cent, while looking forward hopefully to the day when we have so well solved the many problems of project evaluation that we can seriously face up to trying to distinguish 10 per cent yields from those of 9 or 11 per cent.

(Arnold C. Harberger, 1964, p. 1)

10.1 INTRODUCTION

This is a chapter on applications of the criteria and rules derived previously. In this book we have approached cost–benefit analysis as a set of reasonable shortcuts in the search for the social profitability of projects in a wide sense. There are many possible candidates for cost–benefit analysis.

The economic evaluation of transport infrastructure is the content of sections 10.2 and 10.3. Investment in high speed rail infrastructure is costly, irreversible and subject to cost and demand uncertainty. A cost–benefit analysis of high speed rail is quite similar to road, port or airport evaluation, with time savings, increase in quality, reduction in congestion and the willingness to pay of generated trips as its main benefits. It is a typical investment that can be analysed within a cost–benefit analysis framework without major difficulties and the practical approach can be generalized to any other infrastructure.

A controversial policy is the change of ownership of public firms, either understood as the sale of public assets or, as it is widely applied around the world, as concession contracts through which the public sector concedes the provision of public services to the private sector for a predetermined period of time, according to the conditions established in the contract agreement. Section 10.4 presents a basic model, which is easy to apply to the privatization of public firms, and section 10.5 gives an application to a
water supply concession contract. The analytical framework is applicable to the evaluation of price or quality regulation.

The content of section 10.6 is not an application but it is critical for the application of cost–benefit analysis: the importance of incentives. This is a topic that has not received sufficient attention in the economic evaluation of projects, though it is essential to the success of cost–benefit analysis as a tool for informed decision making. The importance of incentives in institutional and contract design and the loss of value of cost–benefit analysis when the objective functions of social agents are overlooked.

Ex ante evaluation may become an empty bureaucratic procedure unless we understand the institutional context and the conflicting objective functions of the agents involved. The understanding of the different levels of government usually involved in major projects and policies, as well as the implication of a menu of contracts for private participation, is a fundamental step if we want to avoid the conversion of cost–benefit analysis into a useless administrative procedure instead of an economic tool for rational decision taking.

10.2 INVESTMENT PROJECTS: ECONOMIC EVALUATION OF INFRASTRUCTURE

Investment in high speed rail has won the support of its direct users, who value its high quality and speed; of governments, who see it as an instrument for integration and for the reduction of congestion on roads and at airports; of railway authorities, for which it has been a path of renewal, in a context of railways’ declining market share in the distribution of traffic between modes of transport; and finally of the industrial firms producing railway equipment, for the volume of business that it involves.

The introduction of the technology known as high speed rail, consisting of infrastructure and rolling stock that allows the movement of passenger trains at 350 km/hour, has led to a revival of rail transport. Apart from the industry propaganda and the myth of high speed trains, this technology competes with road and air transport over distances of 400–600 km, in which it usually is the main mode of transport. For short distance trips, the private vehicle recovers the market share, and for long distance travel, air becomes the hegemonic mode of transport.

The fundamental problem of high speed is not technological, but economic: the cost of high speed rail infrastructure is high, sunk and associated with strong indivisibilities (the size of the infrastructure is virtually the same for a line regardless of the volume of existing demand). In corridors
with low traffic density, the cost per passenger is very high, which makes the financial stability unfeasible.

As investment costs in high speed infrastructure are well above those required by conventional trains, and its use is associated with very pronounced decreasing average costs, population density largely determines the financial and social viability of the investment. In this section we present a simple model for assessing high speed rail investment. Regardless of market shares and the political rhetoric about its role in territorial integration and regional development, we intend to answer the following question: is society willing to pay for the social cost of high speed rail?

Cost–Benefit Analysis of High Speed Rail Investment: A Basic Framework

Although the effects of building high speed rail infrastructure are many, the first direct effect is the reduction of travel time (while simultaneously increasing the quality of travel) and, when cross effects are significant, the reduction of congestion in roads and airports.

In cases where the saturation of the conventional rail network requires capacity expansions, the construction of a new high speed line has to be evaluated as an alternative to the improvement and extension of the conventional network, with the additional benefit of releasing capacity. Obviously the additional capacity has value when the demand exceeds the existing capacity on the route. In these circumstances the additional capacity can be valuable not only because it can absorb the growth of traffic between cities served by the high speed railway but also because it releases capacity on existing lines to meet other traffic, such as suburban or freight.

The generated traffic is a direct benefit for the users, which is generally valued as half of the benefits of existing users according to the ‘rule of a half’ (see Chapter 2). However there is a debate over whether this generated traffic involves greater economic benefits that are not captured in conventional cost–benefit analysis. Leisure travel and business trips can benefit the destination, though it is crucial to distinguish whether it is a genuine expansion of economic activity or a simple relocation of jobs and previous economic activity.

The debate on these issues focuses on whether these changes are additional economic activity or a simple relocation of the pre-existing activity. Besides, many indirect benefits are associated with investment in transport infrastructure in general and not exclusively in high speed, so even if they increase the social return on the investment in transport, they do not necessarily place high speed in a better position over other options for transport investment. Moreover, in undistorted competitive markets, theory tells us that there the net benefit of marginal change in congestion is zero.
Regarding the spatial effects, high speed lines tend to favour central locations, so that if the aim is to regenerate the central cities, high speed train investment could be beneficial. However, if the depressed areas are on the periphery, the effect can be negative. The high speed train can also allow the expansion of markets and the exploitation of economies of scale, reducing the impact of imperfect competition and encouraging the location of jobs in major urban centres where there are external benefits of agglomeration (Graham, 2007). Any of these effects are most likely to be present in the case of service industries (Bonnafois, 1987) (see Chapter 3).

The environmental impact of investment in high speed rail points in two directions: one of them is the reduction in air and road traffic. In such cases its contribution to reducing the negative externalities of these modes could be positive, though we must not forget that it requires a significant deviation of passengers from these modes. Moreover, the use of capacity must be high enough to offset the pollution associated with the production of electric power consumed by high speed trains as well as noise pollution. Rail infrastructure also has a negative environmental impact such as the barrier effect as well as the land taken for the access roads needed for construction and subsequent maintenance and operation. The net balance of these effects depends on the value of the affected areas, the number of people affected, the benefits of the diverted traffic and so on (Nash, 2009).

The assessment of the social profitability of high speed rail requires us to consider this public action as an investment in fixed infrastructure and specialized rolling stock, which incurs for its operation thereafter costs of maintenance, energy, materials and labour, some fixed and others dependent on the volume of demand. The initial investment and the annual costs allow us to obtain a flow of benefits over the life of the infrastructure.

The costs of building the railway infrastructure and the subsequent maintenance and operation costs can be expressed in a simplified form as follows:

\[
TC = I_0 + \sum_{t=0}^{T} \frac{(C(x_t) + C_t)}{(1 + i)^t},
\]

where:
- \(TC\): total costs
- \(I_0\): investment costs in year 0
- \(C(x_t)\): annual operating costs dependent on \(x_t\)
- \(C_t\): annual fixed costs of maintenance and operation in year \(t\)
- \(T\): project life
- \(i\): social discount rate
- \(x_t\): passenger trips in year \(t\).
Applications

For simplicity we assume here that the benefits of high speed rail investment are limited to time savings and quality improvements. Indirect effects and the net balance of environmental impacts are considered irrelevant.

The investment in high speed rail would be socially profitable if its benefits outweigh its costs, requiring expression (10.2) to be greater than zero:

$$NPV = I_0 + \sum_{i=1}^{T} \frac{(B(x_i) - C(x_i)) (1 + \theta)^{t-1} - C_i}{(1 + i)^t}, \quad (10.2)$$

where $B(x_i)$ are the annual benefits and $\theta$ is the annual growth rate of annual net benefits.

Given the indivisibilities affecting high speed infrastructure, the values of $I_0$ and $C_i$ are not very sensitive to the volume of demand (for a given line length). The higher the values of these parameters, the harder it is to reach a positive $NPV$. This also applies to the variable cost, although this cost depends on the demand volume. The key is therefore the benefits that, for a given $T$, depend on the number of users during the life of the project. The level of demand (the initial volume and its growth rate) appears as the basic determinant of satisfying the condition of social profitability in (10.2).

The importance of demand can be illustrated by Figure 10.1, which represents the function of the average cost ($AC$) and marginal cost ($MC$) of a high speed train, whose total cost function is $C = K + cx$, where $K$ is the annualized fixed cost, $c$ is the marginal cost per passenger trip and $x$ the number of passenger trips per year (constant over the life of the project). The graph shows two demand curves: the $D_1$ curve for a country of low...
population density, and $D_2$, corresponding to a country with high population density (assume both have the same level of income per capita).

Assuming that the price is equal to the marginal cost, the revenues are lower than the total costs in both countries, requiring public funding to ensure financial stability. In the cases shown in Figure 10.1 the fixed costs are not covered. However the cost–benefit analysis of both projects shows very different social profitability.

In the case of a high demand country ($D_2$), the revenues $cgx_20$ only cover the variable costs and losses are represented by the area $hfgc$, representing the annual fixed cost; however the willingness to pay of individuals exceeds the cost of the train: the area $\gamma_2gx_20$, which is the total value of the high speed rail to the individuals (assuming that there are no indirect effects or externalities, or any other distortions), allows the variable costs $cgx_20$ to be covered. The resulting consumer surplus $\gamma_2gc$ (which we might call the willingness to pay for capacity) exceeds the cost of that capacity $hfgc$ since $\gamma_2eh$ is greater than $efg$. The economic result is positive despite the financial result being negative. In this case, the high speed rail infrastructure increases social welfare.

In the case of the low demand country ($D_1$) the financial result is negative and equal to the area $abdc$; however, unlike in the country with high demand, the inclusion of the consumer surplus as a benefit of the project does not allow positive results. The willingness to pay for capacity in this country ($\gamma_1dc$) is less than the cost of capacity ($abdc$). The economic evaluation shows an annual social loss equivalent to the area $abdt_1$, so the conclusion is that the country with demand $D_1$ should not build the high speed infrastructure.

It is worth recalling here the assumptions on which the conclusions regarding the desirability of building the infrastructure rest. The first is that there are no other relevant effects; the second is that there are no budgetary constraints that would increase the opportunity cost of public funds required and therefore would reduce the social returns; the third is that we are evaluating a project in isolation and the decision is of the ‘accept–reject’ type, hence an $NPV > 0$ is a sufficient condition to accept the project; and the fourth is that efficiency is the only decision criterion.

## 10.3 COST–BENEFIT ANALYSIS OF HIGH SPEED RAIL: AN ILLUSTRATION

Let us consider an investment project in a new high speed rail line of 500 km without intermediate stations. This line would be built in a corridor
where conventional rail, road and air transport are in operation. The lifespan of the project is 30 years.

The investment costs amount to $4500 million for the base year of the project (year 0); the residual value is zero, and the average variable cost of the high speed train is constant and equals $45 per passenger trip. The ticket price of the high speed train is set at $55. The average costs of the conventional train, car and plane are also constant and equal to $36, $50 and $90 respectively, and in all three cases the price is equal to the average cost. The generalized price of travel for each transport mode is given by $g_i = p_i + v_i t_i$ ($i =$ train, car and air). The travel times $t_i$ for each mode and time values $v_i$ are listed in Table 10.1.

The time values in Table 10.1 are identical for all the users within each mode, and they grow every year by the same proportion as income. The interest rate equals the social rate of discount (5 per cent). All the values are expressed in real terms.

There is uncertainty about the volume of demand for the project; hence, for illustrative purposes, we consider two scenarios for predicting future demand. The first scenario is pessimistic, with an annual volume of 3 million passenger trips for the first year of the project, while the second scenario is optimistic, with a volume of 6 million passenger trips for the initial year. We assume an elasticity of demand with respect to income of 1.2 and an income growth rate of 2.5.

In Chapter 2 we discussed two equivalent approaches for conducting cost–benefit analysis: the first, adding the changes in the surpluses of the social agents; the second, calculating time savings, the benefits of the generated traffic, the costs of accidents avoided and the change in costs. We assume that no other benefits or costs exist.

### Change in Social Surplus

Following the first procedure we must add the surpluses of the various social agents. First the user gains in terms of generalized price (money

<table>
<thead>
<tr>
<th></th>
<th>High speed train</th>
<th>Conventional train</th>
<th>Car</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total travel time</strong></td>
<td>2.67</td>
<td>6.67</td>
<td>5.30</td>
<td>2.58</td>
</tr>
<tr>
<td>(in decimal terms)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Value of time ($)</strong></td>
<td>–</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>(in year $t = 1$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
plus time), calculated separately for each group of users of other modes of transport that shifts to the high speed train. The introduction of high speed implies changes in the generalized price by mode as reflected in Table 10.2.

From these data on the changes in the generalized price per passenger trip we can evaluate the benefits for the users of the high speed train. Previously we had to distinguish between those passenger trips changing mode (whose consumer surplus is obtained as the difference in generalized prices) from generated passenger trips. In the latter case the increase in consumer surplus generated with the introduction of high speed is equal to the triangle bounded by the demand curve and the price and quantities differences. The benefits are calculated as half of the difference in the generalized costs, with and without the project, multiplied by the generated passenger trips.

Table 10.3 shows, for the low demand scenario (3 million passenger trips), the incoming users from different modes and the generated traffic (the same proportions are assumed for the case of high demand) and the benefits that these users obtain in the first year.

In the first year of the project the costs and benefits are:

- Time savings and generated traffic: 58,087,500
- Reduction in accidents: 1,800,000
- Revenue: $55 \times 3,000,000 = 165,000,000$
- Operating costs: $45 \times 3,000,000 = 135,000,000$

The results obtained for the whole project in both demand scenarios are as shown in Table 10.4.

### Changes in Willingness To Pay and Resources

The second approach to calculating the \(NPV\) of the project is to focus on the changes in the willingness to pay and the resources, ignoring transfers.
Figure 10.2 shows, for the case of the conventional train in the low demand scenario, the benefits of time savings for existing users (the dark shaded area).

In Figure 10.2 the benefits of generated trips are represented by the light shaded area. It can be seen how, for this generated traffic, the benefits are obtained by calculating the willingness to pay for the new trips (the area under the demand function in the segment of the 648,000 generated trips) minus the total value of the time spent on such trips (the operating costs of attending this traffic are not represented in the figure and have to be deducted to obtain the benefit of generated traffic).

Similarly, considering separately the benefits of the existing passengers’

Table 10.3  First year users’ benefits (low demand) (values in $)

<table>
<thead>
<tr>
<th></th>
<th>Users’ benefits in the deviated demand group</th>
<th>Users’ benefits in the generated demand group</th>
<th>Total benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional train</td>
<td>$1,500,000 \times 21 = 31,500,000$</td>
<td>$0.5 \times 648,000 \times 21 = 6,804,000$</td>
<td>$38,304,000$</td>
</tr>
<tr>
<td>Car</td>
<td>$300,000 \times 21.3 = 6,390,000$</td>
<td>$0.5 \times 126,000 \times 21.3 = 1,341,900$</td>
<td>$7,731,900$</td>
</tr>
<tr>
<td>Air</td>
<td>$300,000 \times 33.2 = 9,960,000$</td>
<td>$0.5 \times 126,000 \times 33.2 = 2,091,600$</td>
<td>$12,051,600$</td>
</tr>
<tr>
<td>Total</td>
<td>$47,850,000$</td>
<td>$10,237,500$</td>
<td>$58,087,500$</td>
</tr>
</tbody>
</table>

Table 10.4 Social and financial benefits of the project (millions of $, discounted values)

<table>
<thead>
<tr>
<th></th>
<th>Low demand $q = 3,000,000$</th>
<th>High demand $q = 6,000,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time savings and generated traffic</td>
<td>2013</td>
<td>4027</td>
</tr>
<tr>
<td>Accidents reduction</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>Revenue</td>
<td>3617</td>
<td>7233</td>
</tr>
<tr>
<td>Operating costs</td>
<td>$-2959$</td>
<td>$-5918$</td>
</tr>
<tr>
<td>Infrastructure costs</td>
<td>$-4500$</td>
<td>$-4500$</td>
</tr>
<tr>
<td>Social NPV</td>
<td>$-1792$</td>
<td>$879$</td>
</tr>
<tr>
<td>Financial NPV</td>
<td>$-3842$</td>
<td>$-3185$</td>
</tr>
</tbody>
</table>

Note: Project life = 30 years; income growth rate = 2.5%; discount rate = 5%; income-demand elasticity = 1.2; income-value of time elasticity = 1; $p = 55$. 

Figure 10.2 shows, for the case of the conventional train in the low demand scenario, the benefits of time savings for existing users (the dark shaded area). 

In Figure 10.2 the benefits of generated trips are represented by the light shaded area. It can be seen how, for this generated traffic, the benefits are obtained by calculating the willingness to pay for the new trips (the area under the demand function in the segment of the 648,000 generated trips) minus the total value of the time spent on such trips (the operating costs of attending this traffic are not represented in the figure and have to be deducted to obtain the benefit of generated traffic).

Similarly, considering separately the benefits of the existing passengers’
trips and the new ones, the calculations are made for road and air transport. For the benefits obtained in this way we must add those benefits resulting from a cost reduction in the transport modes that lose traffic and the benefits of avoided accidents. Finally one must subtract the costs of the

Figure 10.2  Benefits of the high speed train (conventional train users)

Table 10.5  Benefits of high speed rail (millions of $, discounted values)

<table>
<thead>
<tr>
<th></th>
<th>High demand</th>
<th>Low demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time savings from deviated trips</td>
<td>2085</td>
<td>4170</td>
</tr>
<tr>
<td>Train</td>
<td>1858</td>
<td>3715</td>
</tr>
<tr>
<td>Car</td>
<td>244</td>
<td>488</td>
</tr>
<tr>
<td>Air</td>
<td>−17</td>
<td>−33</td>
</tr>
<tr>
<td>Benefits of generated trips</td>
<td>1440</td>
<td>2881</td>
</tr>
<tr>
<td>Cost savings</td>
<td>2105</td>
<td>4209</td>
</tr>
<tr>
<td>Train</td>
<td>1184</td>
<td>2367</td>
</tr>
<tr>
<td>Car</td>
<td>329</td>
<td>658</td>
</tr>
<tr>
<td>Air</td>
<td>592</td>
<td>1184</td>
</tr>
<tr>
<td>Accidents</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>Investment costs</td>
<td>−4500</td>
<td>−4500</td>
</tr>
<tr>
<td>Maintenance and operating costs</td>
<td>−2959</td>
<td>−5918</td>
</tr>
<tr>
<td>NPV (social)</td>
<td>−1792</td>
<td>879</td>
</tr>
<tr>
<td>NPV (financial)</td>
<td>−3842</td>
<td>−3185</td>
</tr>
</tbody>
</table>

Note: Project life = 30 years; income growth rate = 2.5%; discount rate = 5%; income-demand elasticity = 1.2; income-value of time elasticity = 1; \( p = 55 \); low demand (first year) = 3,000,000; high demand (first year) = 6,000,000.
Applications

10.4 POLICY EVALUATION: COST–BENEFIT ANALYSIS OF PRIVATIZATION

The economic evaluation of a privatization policy has three key elements: first, the existence of cost differences between the public and the private sector; second, the type of market where the company operates and the ability to exercise monopoly power; and third, the sale price.

Even assuming that the privatization policy results in net benefits for the society, these benefits may not be well distributed. It may be that producers are better off with privatization, but consumers and workers are worse off. Therefore, to assess a privatization project in practice, the practitioner should make a detailed assessment of winners and losers. This will be necessary to design the compensation mechanisms, for reasons of equity or to make the project politically acceptable.

In this section we present a basic model and an application to assess whether the society wins or loses with the privatization of a public firm or service. The privatization model can equally be applied to the evaluation of privatization in the strict sense or a concession, and can also be extended to the evaluation of price and quality regulation.

Social Benefits of Privatization and Sale Price

Often the success of privatization has been associated with the sale price achieved. It is possible that the emphasis on the financial aspects of privatization has made us forget that in the sale of public assets, unlike a transaction between private agents, it is necessary to know what happens after the exchange; for example, a high sale price for the public company may
Introduction to cost–benefit analysis

simply be reflecting the net present value of monopoly profits expected by the buyer of the company.

Let us consider, as a starting point, the case of the public firm represented in Figure 10.3, and the evaluation of the project consisting of its privatization.3

The public company charges a price $p_g$ and produces at the unit cost $c_g$, its annual profit is therefore equal to $(p_g - c_g)x_g$, equivalent to the area $p_gabc_g$ in Figure 10.3. When the company is public the variables are identified by the subscript $g$ (government) and when it passes into private hands by the subscript $p$ (private).

How much is the maximum a private entrepreneur would be willing to pay for the public company represented in Figure 10.3 if there is no change in either the price ($p_p = p_g$) or the cost ($c_p = c_g$)? Assuming that the lifespan of the firm is $T$ years, and the rate of discount is zero for both the public and the private sector, the private value of the company in private hands ($V_{pp}$) can be expressed as:

$$V_{pp} = T(p_p - c_p)x_p.$$  \hfill (10.3)

If the entrepreneur pays $V_{pp}$ for the firm, the public sector gains the discounted benefits generated by the company in private hands during his lifetime $T$. If we call $Z$ the price paid by the entrepreneur to the government for the privatization, the highest possible value of $Z$ is $V_{pp}$.

It should be recalled here that, unless money has a higher value in the hands of the government than in private hands, $Z$ is a mere transfer of income and therefore the government would be indifferent to the value of $Z$. Hence we assume that the value of money in public hands ($\lambda_g$) is higher than the value of money in private hands ($\lambda_p$), $\lambda_p$ having a value equal to

---

Figure 10.3  Privatization without a change in either price or cost
unity, and \((\lambda_g - \lambda_p)\) is the additional value of a dollar when it passes from private to public hands.\(^4\)

Returning to Figure 10.3, for the social surplus to increase with the privatization, the following condition must hold:

\[
\lambda_g Z - \lambda_p Z - \lambda_g T \pi_g + \lambda_p T \pi_p > 0,
\]

where \(\pi_g\) and \(\pi_p\) are the benefits before and after the privatization (i.e. in public and private hands, respectively).

Expression (10.4) shows the changes that occur with the sale of the public company, valued with shadow prices of the money held by the government and the private sector. It can be seen how \(Z\) is received by the government and paid for by the private buyer. Also, the government loses the annual profit obtained before the sale \((\pi_g)\) and, by moving the company to the private sector, the buyer obtains \(\pi_p\). Since we have assumed in Figure 10.3 that the price and the cost do not change, we know that \(\pi_g = \pi_p = \pi\), and (10.4) can be expressed as:

\[
(\lambda_g - \lambda_p) (Z - T \pi) > 0,
\]

the necessary condition to increase the social surplus that cannot be met, since \(Z\) cannot be greater than \(V_{pp} = T \pi\).

Given the situation represented in Figure 10.3, if the government manages to sell the company at the highest price the buyer is willing to pay, privatization does not change the social surplus. If the entrepreneur pays a price below \(T \pi\) the society loses from the privatization.

Suppose now that, as shown in Figure 10.4, the private firm is more efficient than the public enterprise \((c_p < c_g)\) and the price does not change (hence, \(x_p = x_g\)) By privatizing, the producer surplus increases by \((c_g - c_p) x_p\) each year and therefore the private value of the company in private hands \(V_{pp}\) increases by \(T\) times this amount. The increase in \(V_{pp}\) may or may not be translated to an increase in \(Z\), depending on the procedure chosen for the sale. What seems clear is that \(T(c_g - c_p) x_p\) (\(T\) times the area \(c_g b d c_p\)) is the efficiency gain resulting from privatization.

To assess the change in social surplus from the sale of public companies, we can use the following expressions:

\[
\Delta SS = V_{sp} - V_{sg} + (\lambda_g - \lambda_p) Z,
\]

\[
V_{sp} = T(S_p + \lambda_g \pi_p), \quad (10.6)
\]

\[
V_{sg} = T(S_g + \lambda_g \pi_g), \quad (10.7)
\]
where:

$\Delta SS$: change in social surplus

$V_{sp}$: social value of the firm in private hands

$V_{sg}$: social value of the firm in public hands

$S_p$: consumer surplus when the firm is private

$S_g$: consumer surplus when the firm is public.

Expression (10.6) presents the change in social surplus as the difference between the social value of the company in private hands and the social value of the company in public hands plus a proportion of the sale price. We cannot forget that the sale price is a transfer of income between the private buyer and the government (taxpayers) and, unless the money in the hands of government is assigned a higher weight, the sale price does not affect welfare.

Substituting (10.7) and (10.8) into (10.6), considering that as the price is kept constant, $S_g = S_p = S$ (area $\gamma ap_g$) and assuming that the firm is sold for the maximum value that the buyer is willing to pay ($Z = V_{pp} = T\pi_p$), we obtain the social benefits of privatization, in terms of surpluses:

$$\Delta SS = T(S + \lambda_p \pi_p - S - \lambda_g \pi_g) + (\lambda_g - \lambda_p) T\pi_p. \quad (10.9)$$

Simplifying (10.9) we obtain the maximum social value that can be obtained from privatization:

$$\Delta SS = T\lambda_g (\pi_p - \pi_g). \quad (10.10)$$

The expression in brackets in (10.10) is equal to $c_g b d c_p$ in Figure 10.4. The maximum potential social value of privatization is obtained when
the efficiency gain achieved \( \pi_p - \pi_g \) is multiplied by the \( T \) years of the project, multiplied by the shadow price of money in government hands. If the sale procedure fails to transfer efficiency gains to the government, the social surplus will be smaller despite the annual efficiency gain remaining \( c_g b d c_p \). We can also see why, given that there is no change in the consumer surplus, the area \( \gamma a p_c \) does not appear in equation (10.10).

The government has two objectives with privatization according to (10.6): to sell the public enterprise to the bidder that generates the maximum increase in social surplus, and to obtain the highest possible price from this bidder.\(^5\)

Previously we assumed that the price did not change after privatization and therefore the consumer surplus remained constant \( (S_p = S = S) \). We now abandon this assumption and consider the general case in which prices, costs and quantities change after privatization, the discount rate is positive and the privatized company returns to the government some of its profits as taxes.

The social values of the company in the public sector and in the private sector are, respectively:

\[
V_{sg} = \sum_{t=0}^{T} \frac{1}{(1+i)^t} [S_g(t) + \lambda_g \pi_g(t)], \quad (10.11)
\]

\[
V_{sp} = \sum_{t=0}^{T} \frac{1}{(1+i)^t} [S_p(t) + \lambda_p \pi_p(t) + (\lambda_g - \lambda_p) Y(t)], \quad (10.12)
\]

where:

\( Y(t) \): post-privatization taxes

\( i \): social discount rate (we assume it equals the interest rate).

The change in social surplus is obtained by substituting (10.11) and (10.12) into (10.6):

\[
\Delta SS = \sum_{t=0}^{T} \frac{1}{(1+i)^t} [S_p(t) - S_g(t) + \lambda_g \pi_g(t) + (\lambda_g - \lambda_p) Y(t)]
- \lambda_g \pi_g(t) \big] + (\lambda_g - \lambda_p) Z. \quad (10.13)
\]

Operating in (10.13), and adding and subtracting \( \lambda_g \pi_p \), we obtain:

\[
\Delta SS = \sum_{t=0}^{T} \frac{1}{(1+i)^t} [S_p(t) - S_g(t) + \lambda_g (\pi_p(t) - \pi_g(t))]
- (\lambda_g - \lambda_p) \left[ \sum_{t=0}^{T} \frac{1}{(1+i)^t} (\pi_p(t) - Y(t)) - Z \right]. \quad (10.14)
\]
It is quite difficult for the government to make the buyer pay a price $Z$ that is equal to the maximum that the buyer is willing to pay; that is, the private value of the company in private hands ($V_{pp}$). This value is equal to:

$$V_{pp} = \sum_{t=0}^{T} \frac{1}{(1 + i)^t} (\pi_p(t) - Y(t)).$$  \hspace{1cm} (10.15)

Substituting (10.15) into (10.14) and expressing the differences in surpluses as increases, we have an operational expression to calculate the social benefits of privatization:

$$\Delta SS = \sum_{t=0}^{T} \frac{1}{(1 + i)^t} [\Delta S(t) + \lambda_s \Delta \pi(t)] - (\lambda_s - \lambda_p) (V_{pp} - Z).$$ \hspace{1cm} (10.16)

For privatization involving an increase in social surplus, expression (10.16) must be greater than zero. The economic interpretation of (10.16) is as follows: for the typical case $\lambda_s > \lambda_p$, and assuming that the government sells the company at the maximum price the buyer is willing to pay ($Z = V_{pp}$), the social benefit of the privatization equals the value of the sum of the change in consumer and producer surpluses, the latter multiplied by the shadow multiplier of money in government hands.

Let us examine more closely expression (10.16). For a given discount rate and assuming (for simplicity) that, after the sale, both the cost and the price are lower, the maximum social benefit to be derived from privatization is in the expression in brackets: the increase in consumer surplus and the change in profits by reducing costs (this last benefit is fully transferred to the government, and so it appears multiplied by $\lambda_s$) when the price of the sale equals the maximum willingness to pay of the buyer. When the sale price $Z$ is less than the willingness to pay for the firm ($V_{pp}$), the social benefit decreases relative to the maximum included in brackets, in proportion to the deviation that occurs ($V_{pp} - Z$).

### Welfare Effects of Price Regulations

Another mechanism that affects the results is the pricing policy post-privatization. When the company operates as a monopoly, its price tends to be regulated. If the firm is awarded to a private entrepreneur under concession, it is usual to introduce procedures for the regulation of prices. The potential efficiency gains from privatization become social benefits when they go to the consumers through lower prices or higher quality, or they become a benefit of the private firm, or go to the government as the result of a well designed sale.
If we set a price equal to the marginal cost \( (p = c_p) \), where \( c_p < c_c \), the efficiency gains increase with the increase in the quantity sold, with the consumers as beneficiaries of the privatization and, unless the shadow price of public funds is sufficiently high and the elasticity of demand sufficiently low, welfare is improved in relation to the situation where the price is unchanged.

The previous argument introduces an implicit assumption that is not credible. First, if the buyer knows that the price is going to be equal to the marginal cost, why buy the company? Moreover why should the entrepreneur try to reduce costs if the regulator sets a price equal to the cost?

A price regulation scheme that prevents this problem of incentives is to introduce a maximum price (price-cap), for example \( p_p = \kappa p_g \), where \( \kappa \) is less than one and higher than the expected proportion of the reduction in the unit cost. Now the cost of the private firm does not appear in the regulatory mechanism, and the company has, in principle, incentives to lower costs because the price is fixed \( (\kappa p_g) \) and therefore, if the entrepreneur manages to reduce the cost below the regulated price, it will increase its profit.

A combination of price regulation schemes based on incentives, with an auction design that maximizes the revenue from the sale of the company, is the way to maximize the social surplus if the market is not competitive. The privatization of services like water supply, electricity or public transport is a good example.

10.5 COST–BENEFIT ANALYSIS OF THE CONCESSION OF A RESIDENTIAL WATER SUPPLY

In this section we evaluate a policy of privatizing (through a concession contract) the water supply in a medium sized city. The aim is to show how the theoretical framework of the preceding sections can be applied to real data on costs and prices in an actual market. The methodology is equally applicable to any other public service.

To calculate the increase in social surplus we need to compute the expression (10.16). This requires information about the values of \( \lambda_v, \lambda_p \) and \( V_{pp} \), and also about the water demand. We will make the following simplifying assumptions:

- **Assumption 1.** The shadow price of money in public and private hands is equal to one: \( \lambda_v = \lambda_p = 1 \). This assumption eliminates the last term of expression (10.16), which could also be achieved by assuming that...
the winning bidder pays the maximum \( Z = V_{pp} \). The assumption \( \lambda_g = \lambda_p = 1 \) has the additional advantage of implying equal weights for changes in consumer surplus and profits in expression (10.16).

- **Assumption 2.** The water demand is linear and therefore we apply the ‘rule of a half’ to calculate the consumer surplus. Using this assumption the changes in consumer surplus and profits are:

\[
\Delta S(t) = \frac{1}{2}(p_g(t) - p_p(t))(x_g(t) + x_p(t)). \tag{10.17}
\]

\[
\Delta \pi(t) = (p_p(t) - c_p(t))x_p(t). \tag{10.18}
\]

- **Assumption 3.** The average cost is lower when the firm is in private hands \((c_p < c_g)\). The changes in consumer surplus and profits are constant over time:

\[
\Delta S(t) = \Delta S; \Delta \pi(t) = \Delta \pi. \tag{10.19}
\]

- **Assumption 4.** There are no subsidies. The public company covers costs \((p_g = c_g)\), while regulated private companies are allowed to set \(p_p = kc_p\), where \(k \geq 1\) is determined by the regulator. This assumption is imposed to abstract the problem discussed here – the evaluation of the increase in welfare through a change of ownership – from considerations of optimal pricing and optimal subsidies, plus the problem of asymmetric information.

The fourth assumption allows us to determine the price levels \(p_g\) and \(p_p\) required to evaluate \(\Delta SS\). Relaxing this assumption would imply making additional assumptions about the level of subsidy.

The four assumptions we have made allow us to simplify (10.16) to:

\[
\Delta SS = \sum_{t=0}^{T} \frac{1}{(1 + i)^t} \left[ \frac{1}{2}(p_g - kc_p)(x_g + x_p) + (k - 1)c_px_p \right], \tag{10.20}
\]

where \((k - 1) \geq 0\) is the mark-up (the margin of the price over the cost) that the regulated private firm is allowed to set.

Graphically, if \(D\) is the linear function of water demand represented in Figure 10.5, the terms in brackets in expression (10.20) represent the value of the area \(p_gabcdc_p\). The area \(p_gabdcp_p\) is the change obtained in consumer surplus, while \(p_pabcdc_p\) represents the profits of the company in the case \(k > 1\).

In order to evaluate the change in welfare obtained through privatization, using expression (10.20) the only new values needed are \(p_g, c_p, x_p, i\)
Applications

We know that $p_g$ and $x_g$ are the observed values of the price (equal to the marginal cost in this case) and the production level of the public enterprise.

Table 10.6 shows the basic data of the projects consisting of privatizing the public water supply firm. The concession will last 30 years, the benefits are produced at the end of each year and the terminal value is zero. The rate of discount and the interest rate are equal to 5 per cent. The public

Figure 10.5 Privatization of a residential water supply

Table 10.6 Basic data for the evaluation of the water concession project

| $I$ | 0 |
| $T$ | 30 |
| $i$ | 5% |
| $x_g$ | 300,000 |
| Demand-price elasticity | Triangular probability distribution* ($-0.7; -0.4; -0.2$) |
| Demand-income elasticity | Uniform probability distribution** (0.25; 0.5) |
| $p_g = c_g$ | 2 |
| $p_p$ | $1.1 \, c_p$ |
| $c_p$ | Triangular probability distribution (1; 1.6; 2). This implies a range for cost reduction (%) of 0–50, with 20% as the most likely value |
| Income growth rate | Uniform probability distribution (0.01; 0.03) (each year) |

Notes: * Triangular probability distribution (min., most likely, max.); ** Uniform probability distribution (min., max.).

and $k$. We know that $p_g$ and $x_g$ are the observed values of the price (equal to the marginal cost in this case) and the production level of the public enterprise.

Table 10.6 shows the basic data of the projects consisting of privatizing the public water supply firm. The concession will last 30 years, the benefits are produced at the end of each year and the terminal value is zero. The rate of discount and the interest rate are equal to 5 per cent. The public
company sells 300,000 m$^3$ of water per year at a price (equal to the marginal cost) of $2. The private company will be allowed a mark-up of 10 per cent over its private marginal cost ($c_p$). Although the reduction in the post-privatization cost is unknown, we assume a range with a maximum of 50 per cent and a minimum of zero, and with 20 per cent as the most likely value.

We need to predict the demand post-privatization. There is evidence on residential water demand elasticities (see the meta-analysis by Espey et al., 1997; also see Dalhuisen et al., 2003). Based on this evidence, and after the elimination of outliers and the less likely values, we use a triangular probability distribution with a minimum demand-price elasticity value of −0.7 and a maximum value of −0.2, with −0.4 as the most likely value.

The income grows during the concession period. We assume that the annual rate of growth can be represented by a uniform probability distribution with a minimum value of 1 per cent and a maximum of 3 per cent. We also assume that the demand-income elasticity is within the range 0.25–0.5 but we do not know anything about the likelihood of any particular value, so we use a uniform probability distribution.

Then we have four random variables in the evaluation: two for the demand elasticities, one for the post-privatization cost reduction and one for the annual growth of income. There is an important difference between the last one and the other three. When calculating the \(NPV\), the computer program, according to our model, draws one value from the probability distribution of the income growth rate for each year of the 30 years of the concession period. Hence, for any \(NPV\) value there are 30 independent drawings from the income growth rate probability distribution. This implicitly assumes that the annual growth rate is uncorrelated with previous annual rates.

On the contrary, the other three probability distributions reflect evaluation uncertainty. This means that we do not know the exact values of some parameters, so we tell the program to choose only one value of each probability distribution and keep this value fixed for the \(T\) years in any iteration. The program will choose another value in the next iteration.

Figures 10.6 and 10.7 represent the social and financial probability distributions for the \(NPV\). Both probability distributions support the approval of the policy. The ranges of likely social and financial net present values are positive and the expected values are $2.5 million and $0.8 million for the social and financial \(NPV\) respectively.

Nevertheless the probability distribution of the consumer surplus represented in Figure 10.8 shows that although the expected value is $1.6 million there is an 8.3 per cent probability of a negative consumer surplus of between $1 million and zero. The reason for this segment of negative
values for the consumers rests on the possibility of a 10 per cent mark-up combined with a cost reduction that ranges between 0 and 50 per cent. Any 10 per cent increase in price with insufficient cost reduction can produce an increase in price for the consumer, which in terms of the social \( NPV \) is partially compensated for by an increase in producer surplus. The deadweight loss of efficiency is generally compensated for, in this particular case, by the gain in efficiency thanks to the cost reduction.

One lesson from this case study is that by working exclusively with expected values we miss some valuable information. Risk analysis provides the decision maker with a more complete picture of what is expected.
Another lesson is that too much aggregation hides useful information. In this case the probability distribution of consumer surplus shows that consumers may be losers in this privatization with an 8.3 per cent probability. This may be considered tolerable. Perhaps the regulator could contemplate the introduction of a price cap equal to the existing price before privatization.

10.6 INSTITUTIONAL DESIGN, CONTRACTS AND ECONOMIC EVALUATION

The development of any project involving initial investment costs and maintenance and operating costs in subsequent years requires an estimate of those costs in accordance with the criteria that have been developed in previous chapters. A positive net social benefit in the ex ante cost–benefit analysis does not guarantee the realization of this benefit during the life of the project. Unfortunately it is common to find some projects that should not have been built according to their ex post benefits and costs.7

The explanation for the discrepancy between expected and actual values in the case of infrastructure lies partly in the nature of the engineering works. The fact that the expected benefits do not materialize according to the ex ante cost–benefit analysis is explained by contemplating the characteristics of the activity being evaluated. Transport energy and water infrastructures have a long life (over 30 years), have few alternative uses, are

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**Figure 10.8 Consumer surplus**

Values x 10^{-7}
very expensive and the actual demand tends to deviate from that predicted for such long periods of time.

External shocks caused, for example, by the decline of economic activity, or a reduction in the population of the region where the infrastructure was built, significantly affect the demand of electricity, the highway or the company that supplies water; and thus the social benefit of the project. It might also happen that costs in the construction phase and subsequent maintenance and operation phases experience changes because of unexpected increases in the prices and quantities of the inputs (energy, labour, etc.) required for the normal operation of infrastructures.

These mismatches between predictions and reality are usual and unavoidable in the economic life of a project. Little can be done beyond improving forecasting techniques and risk analysis. However the prediction errors are not the only explanation for the differences between ex ante and ex post economic profitability. The institutional design and the type of contracts used for the construction and operation may enhance or mitigate the negative effects stemming from unanticipated changes in costs, demand and technology on the ex post benefits of the evaluated project, modifying the behaviour of economic agents and affecting the level of effort to minimize costs or the selection of the appropriate technology.

Now we discuss the consequences of the existence of different levels of government with conflicting objectives and their implications for the selection of contracts for private participation. The understanding of these issues for cost–benefit analysis is paramount in a context of asymmetric information and demand and cost uncertainty.

Two Levels of Government and their Effects on the Selection of Projects

The construction of a major infrastructure project is an expensive task. It is an investment that gathers the following characteristics: lumpy, irreversible and costly. The decision to invest public funds in the construction of a dam or a road is subject to cost and particularly demand uncertainty. The irreversibility of the decision makes the economic appraisal of the project quite relevant. Hence it is sensible to examine how the institutional design affects the final choice in the allocation of public funds to these types of projects.

National and supranational governments support the implementation of key infrastructure with public funds. To understand the effects of this public support in the investment decision it is useful to distinguish two levels in the process of decision making and funding of major infrastructure projects. The first one relates to the institutional design, in which supranational and national governments (or national and regional
governments) agree on the projects to be financed. The second one is related to the selection of contracts for the construction and operation of the infrastructure. This level includes the relationship between the national (or regional government) beneficiary of the project with the operator(s) responsible for the construction and operation of the project.8

There are two extreme cases in the menu of contracts: the cost-plus and the fixed-price. In the first case the agency responsible pays the full cost of the project and in the second case it pays an ex ante fixed amount. This characterization of contracts can be applied to funding, price regulation, construction contracts, and so on. The common ‘contract’ between national and regional government for the financing of an infrastructure project is a kind of cost-plus.

The problem with this type of cost-plus financing mechanism is that the public funds a national government obtains from the supranational agency, or the regional government obtains from the national government, increase with the total investment costs and decrease with the net revenues. Hence this financing mechanism penalizes the internalization of externalities, leads to excessive demand and biases the capacity size and the choice of technology.

Let us suppose that a country facing a problem of capacity in the transport network is considering mutually exclusive projects, including the construction of a new high speed rail line that can apply for financial support from a supranational planner that maximizes social welfare. The country is governed by a politician who must decide the main characteristics of the project (let us say high speed rail or an upgraded conventional train), make a cost–benefit analysis and report it to the supranational planner in order to receive funds for the construction of the infrastructure.

The effects of the present system of co-financing in the EU, or any other system in which a national government pays for the infrastructure within the national budget and the regional government decides on the type of project to be financed, can be modelled in the following way (de Rus and Socorro, 2010). Suppose only two periods. During the first period the new rail infrastructure is constructed. During the second period the citizens of the country use it. The real construction costs are paid by the supranational government. We know that actual costs do not necessarily coincide with the minimum investment cost. To minimize construction costs requires the politician to make an effort, which is a cost for him.

It is not uncommon for national governments to be better informed than the supranational agency about the transport problem and the set of alternatives available and therefore about the minimum investment cost required to solve the problem. For this reason we assume that the supranational planner cannot observe (or verify) either the minimum investment
cost or the effort exerted by the politician in order to be efficient. Moreover the national government has to decide the price to be charged for the use of the new infrastructure and consequently the number of users. There are also operating and maintenance costs, which are privately known, and in many cases different technologies and/or capacity sizes with significant cost differences.9

Once we abandon the idea of perfect information and assume that the utility function of the politician (the national government) depends on his own private income (only obtained if the politician is governing the country), we can go further in explaining some of the evidence concerning national government decisions on expensive infrastructure and suboptimal pricing.10 The higher the welfare of voters in the second period, the higher the probability of re-election. The welfare of voters in the second period is the sum of their consumer surplus and the value of social expenditures.

In a world of perfect information, the supranational agency maximizes social welfare, forcing the national government to exert the maximum level of effort and so minimizing project costs and introducing marginal social cost pricing. In the real world, efforts and marginal costs are not observable and the behaviour of the national government will respond to the incentives of the financing mechanism.

With the present financing mechanism (as well as with any other cost-plus financing system) it is costly to be efficient. Governments have no incentives to minimize investment costs or to introduce optimal pricing. There is a bias in favour of expensive latest technology mega-projects, and the pricing will depart from user pays or polluter pays principles as the higher the price for the use of the new national infrastructure, the lower the consumer surplus of voters and the lower the probability of re-election. As a consequence the politician will choose the maximum number of users and will not charge for the external costs.

The evidence supports these conclusions. It is remarkable that national and regional governments have promoted the construction of expensive projects with a demand too low to pass a strict cost–benefit analysis. Forecasting error is not an explanation and strategic misrepresentation has been detected worldwide. Many projects show overcapacity, the use of an excessive and expensive technology given the problem to be solved, cost over-runs and, in general, renegotiations that eventually allow the modification of the contract conditions, extending the construction periods, increasing costs and prices and hence breaking the basic rules of public procurement.

These disappointing results are not completely unexpected. As we have already discussed, national governments are in general better informed than supranational planners about the costs and benefits of the infrastructure projects to be constructed in their own regions, and they do not
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necessarily share the same objectives. Governments may have incentives to manipulate project evaluation to gain more funds from the supranational planner. In a context of asymmetric information and different objectives, the relationship between national governments and supranational planners cannot be modelled in a conventional cost–benefit analysis framework.

In this context cost–benefit analysis lacks its original power as a tool for decision making. The existence of information asymmetries and conflicting interests requires a different approach in which incentives are explicitly accounted for. A fixed-price financing mechanism may provide the necessary incentives to reduce costs and charge the socially optimal price. Moreover, with a ‘fixed-price’ type of financing mechanism, cost–benefit analysis is a very useful tool for governments to allocate the supranational funds in the most efficient way.

The fixed-price mechanism in this context is an ex ante fixed quantity of external funding unrelated to costs and revenue. The idea of a fixed quantity financing mechanism is to make national governments (in the case of supranational funding) or regional governments (in the case of national funding) responsible for insufficient revenues and cost inefficiencies as they receive a fixed amount of funding and are the residual claimants for effort. The incentive to introduce optimal pricing is now high as the costs of inefficient pricing are also borne by the politician.

It is worth emphasizing that by giving national (regional) governments an ex ante fixed amount of funds, the supranational agency (national government) loses its influence over the selection of projects. If supranational (national) governments want to establish infrastructure investment priorities, an intermediate solution is to substitute the cost-plus funding method by an alternative financing scheme based on an ex ante fixed-quantity funding linked to generic objectives such as investing in ‘accessibility’ or ‘minimizing the total social cost of the water supply’ in particular areas, a mechanism that should be dissociated in any case from costs and revenues and the selection of any specific technology. The risk of building a socially unprofitable infrastructure would be dissociated from the public funding mechanism as the selection of the more expensive (and maybe inappropriate) project will now have a completely different opportunity cost for the national (regional) governments.11

Contracts, Incentives and Risk Allocation

Private participation in the construction and operation of infrastructure is carried out, mainly, through fixed term concession contracts. The purpose of the concession system is, first the selection of the most efficient concessionaire among those presented to the public tender; and, second,
obtaining the greatest social benefit possible over the life of the infrastructure, while allowing the concessionaire to cover costs. To achieve both objectives, the selection system and the subsequent regulation should take into account the problems of demand and cost information that characterize the activities under concession.

The fixed-term concession contract is in theory a fixed-price contract but it is a kind of cost-plus in practice, given the widely extended use of renegotiation of this type of contract all over the world (Guasch, 2004). The design of fixed-term contracts allocates the demand risk to the concessionaire and this creates problems concerning the selection of the most efficient bidders and the minimization of operating costs during the life of the concession.

Let us consider the case of an infrastructure with construction costs $I_0$ and maintenance and annual operation costs $C_t$, independent of the number of users $x_t$. In the fixed-term concession system the concession is awarded to the bidder who proposes to charge the lowest price, having previously announced the concession period $T$. Cost coverage requires that the discounted value of the net revenue equals the investment cost:

$$I_0 = \sum_{t=1}^{T} \frac{1}{(1 + i)^t} (p_t x_t - C_t).$$  \hspace{1cm} (10.21)

One version of the fixed-term contract is to fix, ex ante, the concession period and the price to be charged, and let the bidders compete in offering a canon to the public agency. With perfect information on demand, the maximum canon that a bidder will offer equals the present value of expected benefits over the life of the concession. \(^{13}\)

All the existing types of fixed-term concession contract \(^{14}\) share the problem of uncertainty of demand, so it is usual to introduce clauses to guarantee minimum revenue, and contract renegotiations are common throughout the life of the concession. This kind of cost-plus contract has effects on the incentives to minimize costs and on the realization of ex ante social benefit; hence it should be considered in the cost–benefit analysis of projects.

Assuming for simplicity that the discount rate is zero and that $p_t$, $x$, and $C_t$ are constants in expression (10.21), during the life of the concession ($T$), for an entrepreneur to be willing to participate in the competition he should expect that the revenue stream would at least cover the total costs:

$$pxT \geq I + CT.$$  \hspace{1cm} (10.22)

If the annual revenues $px$ are greater than the annual costs $C$, we can determine the value of $T$ that will allow the concessionaire to cover its total costs:
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\[ T_f = \frac{I}{px - C}, \]  

(10.23)

where \( T_f \) represents the duration of the concession determined ex ante by the regulator.

With perfect information on demand, the regulator will have set a value of \( T_f \) and the concessionaire (supposing that there is no strategic behaviour) will have offered the best bid (canon paid, lower price to charge the users, etc.) that makes it possible for expression (10.22) to be satisfied as an equality. Figure 10.9 shows the equilibrium situation.

With the concession term \( T_f \) it can be seen how the total costs are covered by commercial revenue. At the same time Figure 10.9 shows that the compatibility of a fixed concession period and the financial equilibrium of the company rest on unrealistic assumptions: perfect information about demand and efficient behaviour by the concessionaire.

Demand uncertainty is common during the life of the project. One can argue that it is virtually impossible to predict demand accurately over a 30-year horizon. On the cost side there are also information problems. The bidder knows the costs, but the regulator does not (this is called asymmetric information). Therefore, if we abandon the assumption of perfect information and allow the possibility of losses and profits over the life of a long term concession, one must analyse the implications of the type of contract chosen.

Suppose that the concession goes to the bidder that, with the same quality, offers to charge the users the lowest price for a given \( T_f \). The basic
idea behind this auction is that firms interested in obtaining the concession will offer the lowest possible price consistent with their costs. Since there is no information on the costs of rival firms, the efficient bidder (who is identified with the subscript $i$) will try, like others, to increase the probability of winning the contract by offering a price that allows it to receive normal benefits:

$$p_i = \frac{I_i + C_i T_f}{x T_f}.$$  \hfill (10.24)

In expression (10.24) it can be observed that if the level of demand $x$ were known, a lowest price auction with a fixed-term concession $T_f$ would achieve its goal: the concession would be won by the firm that has the lowest total cost to implement the project. However, given the problem of uncertainty about demand $x$ and the various estimates that companies can use when preparing their offers, the award mechanism based on a fixed-term concession does not guarantee that the outcome is the best possible.

Because of factors completely exogenous to the concessionaire, demand may significantly fluctuate over the life of a concession. For simplicity consider that there are only two possibilities for the number of users during the lifetime of the project: high demand ($x_h$) with probability $\pi$ or low ($x_l$) with probability $1 - \pi$.

Since bidders are not sure about future demand they will work with an expected value of demand ($x_e$), which we define as $x_e = \pi x_h + (1 - \pi) x_l$. Once the concession has been awarded and the infrastructure has been built, the demand is $x_h$ or $x_l$ and the company may be in a position of profits or losses.

Figure 10.10 shows a situation where, given a concession term $T_f$, the financial equilibrium would be achieved for the expected volume of demand $x_e$ but not for the situations of high or low demand. In the case of high demand revenues are higher than costs in the distance $af$; and if the demand is low costs are not covered and losses are equal to $fb$. In practice both cases are common and a renegotiation of the concession contract is often required to restore the financial equilibrium.

In the case of inelastic demand, if the demand scenario is low and there is no minimum income guarantee, the adjusting variable is $p$. By authorizing an increase in the price, the revenue function $px_l T$ shifts upward to cut the cost function at point $f$, so the financial equilibrium is ensured. Another possibility is to extend the concession period to $T_f$.

If, on the contrary, the demand is high, the economic profits will be politically undesirable since they would appear to the public as a situation
of unjustified privilege, the reduction in price being an easy adjustment. The fall in price moves $px_h T$ downward, reducing the profits to zero if the movement is carried to point $f$. Another possibility is to reduce the concession period to $T_h$.

This is roughly what has been happening in most fixed-term concessions in the world, especially in cases where demand is lower than expected. In fact this concession contract with virtually assured renegotiation becomes indirectly a rate of return regulation of the private firm that has invested in infrastructure construction and operation, with its negative impact in terms of the incentives to minimize costs.

An additional problem is that with demand uncertainty it is quite likely that the most optimistic bidder will be selected instead of the most efficient. A bidder with high costs could win a contract if its beliefs about the number of users are optimistic enough to offset its cost disadvantage compared with the other firms with lower costs. This is easy to see from expression (10.24), which indicates how companies make their calculations to submit their bids.

Let us consider the case of two firms $i$ and $j$, which compete in an auction awarded to the bid charging the lowest price to users. Assuming that firm $i$ is more efficient than $j$ (i.e. it has lower costs, $I_i + C_i T_f < I_j + C_j T_j$), company $i$ should win the contest. However it is possible that firm $j$ is more optimistic about the expected demand, and this led firm $j$ to make an offer with a lower price ($p_j < p_i$), thereby obtaining the concession. From (10.24), in order for the inefficient company $j$ to win the contract, it would be sufficient that the following condition is satisfied:

---

**Figure 10.10  Financial equilibrium in a concession with unknown demand**
\[
\frac{I_i + C_i T_f}{x_i T_f} > \frac{I_j + C_j T_f}{x_j T_f},
\]

or, equivalently,

\[
\frac{x_j^e}{x_i^e} > \frac{I_j + C_j T_f}{I_i + C_i T_f}.
\]

Expression (10.26) shows that the less efficient firm can win the contract if it is optimistic \((x_j^e > x_i^e)\) enough to offset its cost disadvantage. It can be seen that the condition (10.26) can be met if the inefficient firm \(j\) is inefficient either in absolute terms (both construction and maintenance costs are greater than those of \(i\)) or in relative terms (its total cost \(I_j + C_j T_f\) is higher for a fixed duration, although it might be \(I_j < I_i\) or \(C_j < C_i\)).

In conclusion one can note three major negative economic consequences of the conventional fixed-term concession system based on auctions with bids for the minimum price to charge the users:

- In the absence of perfect information on demand, this type of auction does not guarantee the selection of the firm with the lowest costs since the existence of different beliefs about future demand may cause an inefficient and optimistic firm to win the contract.
- If firms anticipate that it may be possible to renegotiate the concession contract in the future because of changes in the volume of users, the incentives to operate at minimum cost are weak since efforts to operate efficiently can be substituted by negotiation with the regulator requiring a price change to restore the normal benefits.
- With the fixed-term system and inelastic demand, price becomes an accounting adjustment variable, tending to rise when demand is low and decline when demand is high, which can lead to an inefficient use of the infrastructure, contrary to what would be desirable from an economic standpoint. In general when demand is low there will be no pressure on existing capacity, and it would be desirable for the price to encourage the use of infrastructure. When demand is high and congestion problems occur, the price should rise in order to ration the available capacity.

The three consequences this type of contract reduce the ex post NPV of the project. The problems of private participation in infrastructure with fixed-term concession contracts are a result of the economic characteristics of these projects, and especially the uncertainty of demand. High costs, long life and asset specificity, coupled with the inability to predict
the demand for the life of the concession, are the elements that cause the renegotiation of contracts and the loss of the essence of a public tender.

An alternative contract to avoid these problems is to change the mechanism of financial adjustment of the concession system. According to our analysis above, the problem of demand uncertainty translates into uncertainty about the revenue the firm expects from the concession. This is the fundamental point that generates the risk of failure of the concessionaires (in the case of low demand) or of the government reducing prices (in the case where demand is high). One possible solution is for the government to fix the price, the level of quality and the discount rate, and for firms to make offers on the revenue they want to receive during the life of the concession, thus eliminating revenue uncertainty, and letting the contract last as long as it takes to earn the revenue included in the winning bid.15

THINGS TO REMEMBER

- Even assuming that the privatization policy results in net benefits for the society, these benefits may not be well distributed. It may be that producers are better off with privatization, but consumers and workers are worse off. Therefore, to assess a privatization project in practice, the practitioner should make a detailed assessment of winners and losers. This will be necessary in order to design the compensation mechanisms, for reasons of equity or to make the project politically acceptable.

- Working exclusively with expected values may neglect some valuable information. Risk analysis provides the decision maker with a more complete picture as too much aggregation hides useful information.

- It is really impossible to foresee all the circumstances a project has to go through during its lifespan. Ex ante demand and costs do not necessarily coincide with ex post values; hence, the economic evaluation of project has to take into account the facts that explain this divergence.

- Actual demand depends on some unpredictable contingencies but also, to some extent, on decisions taken by economic agents. Contract designs both allocate risk and affect the level of effort exerted by firms to reduce costs. There are two basic types of contracts, the cost-plus and the fixed-price, and different combinations depending on the degree of risk and incentives.

- The effects of public support by supranational agencies on the investment decision taken by national governments vary with the institutional design. The existence of two levels of government, in
which supranational and national governments (or national and regional governments) can modify the incentives in the selection of projects, affects the value of cost–benefit analysis as a tool for informed decision taking.

- Private participation requires the design of contracts. The government wants to select the most efficient contractor and to offer a contract that keeps the firm interested in minimizing costs. To achieve their objectives, the government and the firms have to work in a world of uncertainty and asymmetric information. The type of contract can have profound effects on the ex post net present value of projects.

NOTES

2. Time savings are assumed to be the same within each group of users changing mode.
3. The basic model in this section summarizes the excellent work by Jones et al. (1990).
4. The justification for $\lambda_t > \lambda_p$ is that when the government obtains funds through taxes it imposes an additional burden (deadweight loss of the tax) on the economy and therefore it implicitly gives more value to money in its hands than in the hands of the private sector. Therefore, if with the privatization of the public firm the government obtains funds without distorting the economy, the shadow price of these funds should reflect the additional benefit of avoiding the distortion of raising the same revenue through taxes (see section 4.8).
5. Sticking to the four most common types of auctions (English, Dutch, sealed auction and Vickrey auction) the first objective can be achieved with the English and Vickrey auctions, but the second is not guaranteed by any of the four; and therefore we can expect that in general $Z$ is less than $V_{pp}$ especially when, as happens to be the case, the number of bidders is not very high in a privatization process. For a more detailed discussion on auctions see Klemperer (1999).
6. For an analysis of the design of concession contracts and price regulation see Guasch (2004). The importance of these issues in cost–benefit analysis is crucial, since the actual social $NPV$ arising from the implementation of projects is largely affected by regulatory mechanisms.
7. A major infrastructure project was the Channel Tunnel. It links Britain to the Continent 40 m under the sea. A recent evaluation concludes: 'The cost benefit appraisal of the Channel Tunnel reveals that overall the British economy would have been better off if the Tunnel had never been constructed, as the total resource cost has been greater than the benefits generated' (Anguera, 2006, p. 314).
8. This second level has been widely analysed in the economic literature (Laffont and Tirole, 1993; Bajari and Tadelis, 2001; Guasch, 2004; Olsen and Osmundsen, 2005).
9. Cost overruns are common in large infrastructure projects and it has been demonstrated that the deviation is not only explained by unforeseen events (Flyvbjerg et al., 2003).
10. The implementation of the user pays and the polluter pays principles and the reduction of public expenditure have significant political costs (Sobel, 1998). Downs (1957), Niskanen (1971) and Becker (1983) have often assumed that legislators attempt to maximize electoral support. Even if re-election may not be the primary factor motivating
their legislative behaviour, it is still true that legislators react in predictable ways to the electoral costs and benefits of their choices. Thus legislators will favour actions that increase the probability of re-election over decisions that lower it (Sobel, 1998; Robinson and Torvik, 2005).

11. As is common with fixed-price contracts, some kind of quality regulation may be required.

12. In practice a multicriteria tender is commonly used, with several variables among which the price is included.

13. To simplify the exposition we assume that bidders bid their reservation prices; that is, they present their offers with the most that they are willing to pay, so they earn normal profits.

14. Another method is to set the price and award the concession to the bidder who requests the shortest concession period. This case is actually a variation of the fixed-term concession contract since once the public tender finishes the concession period is as fixed as in the price (or canon) concession system.

15. If the concession period is allowed to be variable instead of fixed (remember that $T_f$ is predetermined in the case of the traditional concession), it would be possible to accommodate situations of high or low demand without the need to renegotiate the contract or having to make unwanted changes in prices. For example, in Figure 10.10, in a situation of low demand ($x_l$) the length of the concession is automatically extended to $T_l$, thus allowing the recovery of the total costs. By contrast, in a case of high demand, in a period $T_h$, such recovery of total costs would have been realized earlier and ending the concession at that time would avoid economic profits. This system was first applied in the UK in the construction and operation of a bridge. Later, it was used in the road concession Santiago–Valparaíso–Viña del Mar in Chile. See Engel et al. (2001) and Nombela and de Rus (2004) for an analysis of variable-term concessions.
11. Microeconomic foundations of cost–benefit analysis

One summer, a colleague asked me why I have not bought a parking permit. I replied that not having a convenient place to park made me more likely to ride my bike. He accused me of inconsistency: As a believer in rationality, I should be able to make the correct choice between sloth and exercise without first rigging the game. My response was that rationality is an assumption I made about other people. I know myself well enough to allow for the consequences of my own irrationality. But for the vast mass of my fellow humans, about whom I know very little, rationality is the best predictive assumption available.

(David Friedman, 1996, p. 5)

11.1 INTRODUCTION

This chapter covers the basic theoretical framework of cost–benefit analysis. The objective is to give a clearer picture of what is behind the concept of social welfare and the decision criteria based on the net present value. The approach is theoretical but neither exhaustive nor extremely technical.

The way from individual preferences to social welfare is the content of section 11.2. Section 11.3 addresses the measurement of producer surplus in the product and factor markets. Section 11.4 analyses the three money measures of changes in individual utility most frequently used to evaluate economic changes: compensating variation, equivalent variation and consumer surplus. Section 11.5 deals with uncertainty.

11.2 FROM INDIVIDUAL UTILITY TO SOCIAL WELFARE

Given the social welfare function in (11.1), assuming independent utilities and taking total differentials, the change in social welfare ($W$) can be expressed as (11.2): the sum of the variations in the utility ($U$) of the $m$ individuals who form the society, weighted by the relative importance the
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society attaches to the utility of each individual (i.e. social marginal utility: $\frac{\partial W}{\partial U_i}$).\(^2\)

$$W = W(U_1, U_2, \ldots, U_m), \quad (11.1)$$

$$dW = \sum_{i=1}^{m} \frac{\partial W}{\partial U_i} dU_i. \quad (11.2)$$

The utility of individuals depends on the quantities of the $n$ goods and services ($x$) available for consumption. In the case of individual $i$:

$$U_i = U_i(x_{i1}, x_{i2}, \ldots, x_{in}). \quad (11.3)$$

Assuming that individual $i$ maximizes utility, subject to the constraint that aggregate expenditure cannot be greater than his income, we have the following expression:

$$\text{Max}_{x_{i1}, x_{i2}, \ldots, x_{in}} U_i(x_{i1}, x_{i2}, \ldots, x_{in}) - \mu_i \left( \sum_{j=1}^{n} p_j x_{ij} - M_i \right), \quad (11.4)$$

where:

$x_{ij}$: quantity of good $j$ consumed by individual $i$
$M_i$: income of individual $i$
$p_j$: price of good $j$
$\mu_i$: Lagrange multiplier.

First order conditions are as follows:

$$\frac{\partial U_i}{\partial x_{ij}} - \mu_j p_j = 0, \quad j = 1, \ldots, n, \quad (11.5)$$

$$\sum_{j=1}^{n} p_j x_{ij} - M_i = 0. \quad (11.6)$$

The economic interpretation of the first order conditions is the following: to maximize utility, the individual allocates his income so that, at the optimum and for the selected goods, the marginal utility of consuming the last unit of the good ($\frac{\partial U_i}{\partial x_{ij}}$) is equal to the disutility of paying the price ($\mu_j p_j$) as $\mu_i$ is the marginal utility of income. Moreover the spending on all the chosen goods exhausts the individual’s income.

Solving for $p_j$ in (11.5):

$$p_j = \frac{\frac{\partial U_i}{\partial x_{ij}}}{\mu_i}. \quad (11.7)$$
At the optimum, the individual consumes additional units of good $j$ until the price is equal to the marginal valuation of the good (the ratio of expression (11.7)). Whenever the marginal valuation of the good is greater than its price, the individual increases the consumption of the good until the budget constraint is binding. It may happen that the price is higher than the marginal valuation of the good for some goods and the individual does not consume them. In this case the first order condition is satisfied as a strict inequality (corner solution).

By differentiating (11.3) we obtain expression (11.8), where the variation in the total utility of individuals depends on the marginal utility of each good and the variation in the quantity consumed of that good:

$$dU_i = \sum_{j=1}^{n} \frac{\partial U_i}{\partial x_{ij}} dx_{ij}. \quad (11.8)$$

Substituting the first order condition (11.5) in (11.8) we obtain the variation in individual welfare:

$$dU_i = \sum_{j=1}^{n} \mu_j p_j dx_{ij}. \quad (11.9)$$

Equation (11.9) contemplates utility as a function of quantities consumed, given the prices of goods and services and the income of the individual. The variation of the utility of individual $i$ depends on the marginal variation in the quantities of the $n$ goods consumed, multiplied by their price (which we assume to be constant) and converted to utilities by multiplying by the marginal utility of income ($\mu_i$).

A useful concept for money measures of changes in individual utilities is the indirect utility function. Individual utility used in its original functional form (11.3) is a function of the quantities of goods and services consumed by the individuals, given prices and income. Going back to the first order conditions (11.5) and (11.6), and obtaining the values of $x$ that maximize the utility function as a function of the vector of prices ($P$) and income ($M$), the indirect utility function is the utility function expressed as $U = U(X(P, M))$, or:

$$V_i(p_1, p_2, \ldots, p_n, M) = U_i(x_{11}(p_1, p_2, \ldots, p_n, M), x_{12}(\cdot), \ldots, x_{1n}(\cdot)). \quad (11.10)$$

Adding the budget constraint to (11.10) we can use the indirect utility function to express the utility maximization as a function of prices and income:
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\[ V_i(p_1, \ldots, p_n, M_i) = U_i(x_{i1}(p_1, p_2, \ldots, p_n, M_i), x_{i2}(\cdot), \ldots, x_{ih}(\cdot)) \]

\[ - \mu_i \left( \sum_{h=1}^{n} p_h x_{ih}(p_1, \ldots, p_n, M_i) - M_i \right). \quad (11.11) \]

At the optimum, differentiating with respect to \( p_j \) and noting that \( V(P, M) = U(X(P, M)) \), we obtain how the utility varies with an infinitesimal variation of price:

\[
\frac{\partial V_i}{\partial p_j} = \sum_{h=1}^{n} \frac{\partial U_i}{\partial x_{ih}} \frac{\partial x_{ih}}{\partial p_j} - \mu_i \left( x_{ij} + \sum_{h=1}^{n} \frac{\partial x_{ih}}{\partial p_j} p_h \right), \quad (11.12)
\]

\[
\frac{\partial V_i}{\partial p_j} = \sum_{h=1}^{n} \left( \frac{\partial U_i}{\partial x_{ih}} - \mu_i p_h \right) \frac{\partial x_{ih}}{\partial p_j} - \mu_i x_{ij} = -\mu_i x_{ij}, \quad (11.13)
\]

because at the optimum (see first order condition (11.5)),

\[
\left( \frac{\partial U_i}{\partial x_{ih}} - \mu_i p_h \right) \frac{\partial x_{ih}}{\partial p_j} = 0; \quad h = 1, 2 \ldots n. \quad (11.14)
\]

The result obtained in (11.13) is well known in microeconomic theory as Roy’s Identity. As we have seen, it is easily derived by applying the envelope theorem.³

By totally differentiating (11.10) we obtain the expression (11.15), where the variation in the total utility of the individual depends on the marginal utility with respect to each change in price multiplied by the price changes, plus the marginal utility of income multiplied by the change in income:

\[
d V_i = \sum_{j=1}^{n} \frac{\partial V_i}{\partial p_j} dp_j + \frac{\partial V_i}{\partial M_i} dM_i. \quad (11.15)
\]

Keeping income constant (\( dM_i = 0 \)) and using the result in (11.13):

\[
d V_i = - \sum_{j=1}^{n} \mu_j x_{ij} dp_j. \quad (11.16)
\]

This expression is equivalent to (11.9) for projects with price changes (keeping quantities constant). In this chapter we only deal with changes either in the quantities of goods (11.9) or in the prices (11.16) without changes in the costs. In Chapter 2 we deal with situations in which prices and quantities change simultaneously.⁴

Expressions (11.9) and (11.16) show how to assess the change in individual utility as a result of a change in quantities and prices. The change
in prices is multiplied by the quantity and the change in quantities is multiplied by prices. To convert this change in utility, expressed in monetary units, we need \( \mu_i \) to be constant and, unless \( \mu_i \) is constant, we cannot directly associate the monetary changes with changes in utility. Identical increases in income result in different changes in utility if the marginal utility of income is not constant.

With utility increasing with income \( (\partial U / \partial M > 0) \) but less than proportionately \( (\partial^2 U / \partial M^2 < 0) \), unless we know the value of the marginal utility of income for individuals with different income levels, it is not possible to convert the aggregate changes in quantities or prices of goods in utility changes.

The above argument is useful for understanding the economic rationale of the measurement of social benefits arising from the implementation of a project. As the utility is not observable, we use monetary measures that reflect the change in the utility of individuals. We are not directly measuring the change in utility, but the change in the willingness to pay or accept, with monetary measures that, despite their limitations, allow us to gain approximations of what we win and lose with the project.

Returning to the social welfare function (11.1) and its total differential (11.2), and using the results derived in this section, we can express the effect of a small change in quantities (11.17) and prices (11.18) on social welfare:

\[
\begin{align*}
dW &= \sum_{j=1}^{n} \sum_{i=1}^{m} \partial W \frac{\partial U_i}{\partial M_i} p_j dx_{ij}, \quad (11.17) \\
&= -\sum_{j=1}^{n} \sum_{i=1}^{m} \partial W \frac{\partial U_i}{\partial M_i} x_{ij} dp_j. \quad (11.18)
\end{align*}
\]

For the economic interpretation of both expressions it is useful to read them from right to left. The project may involve a change in quantities (with \( p \) constant) or a change in prices (with \( x \) constant). In both cases these changes are converted into monetary units, multiplying by price or quantity, respectively.

For such monetary changes to be converted into changes in social welfare there are two types of weights. First, the marginal value of income for each individual \( (\partial U / \partial M) \), which depends on the income level of the individual. This weight transforms income into individual utility. The second weight \( (\partial W / \partial U) \) converts individual utility into social welfare.

For society to improve, with the increase in individual utility, \( \partial W / \partial U_i \) must be positive. The partial derivative \( \partial W / \partial U_i \) can be different for different individuals. The society may give more weight to social groups with different income levels, health conditions or any other relevant
characteristic. The two weights interpreted together are the social marginal utility of income.

Expressions (11.17) and (11.18) aggregate the change experienced by the $n$ individuals, initially measured in monetary units, using the weights described. In (11.17) and (11.18) there is an implicit social welfare function that determines how to proceed with regard to the weighting scheme. The simplest case is the potential compensation, or Kaldor–Hicks criterion, in which benefits and costs are added unweighted (i.e. it is implicitly assumed that the two weights in (11.17) and (11.18) are equal to one). The potential compensation criterion implies that the marginal utility of income is constant, and society gives the same weight to the utility changes of all individuals regardless of income, health status, and so on.

Other approaches are possible with equations (11.17) and (11.18). One possibility is to correct for the marginal utility of income and add the changes in utility, giving the same social value to the utility of any individual. In this way, if the marginal utility of income is decreasing, we will outweigh the benefits and costs of those with lower income. It should be emphasized that this correction does not have any redistributive basis although it may seem so if we compare it with the potential compensation criterion.

Another possibility is a welfare function that introduces the social aversion to inequality. In this case, once the net benefits of each individual are corrected according to their marginal utility of income, it introduces an additional weight (the social marginal utility) that varies inversely with the income of the individual in higher or lower proportions depending on the degree of egalitarianism.

11.3 MEASUREMENT OF PRODUCER SURPLUS

The difference between a firm’s revenues and variable costs of production is called producer surplus. This surplus is the gross profit in the sense that fixed costs have not been deducted. Suppose a company has an annual revenue of 100 and its total costs are equal to 100, of which 50 are variable and 50 fixed. Variable costs are by definition avoidable if the firm produces nothing. Assume that the fixed costs are sunk costs. With this information the firm has zero profits and a producer surplus equal to 50.

Suppose the owner of the company is asked for the maximum he is willing to pay to keep the firm open once production has started. Excluding any strategic consideration his answer is 50. Though the company earns a profit of zero, the producer surplus indicates the maximum he is willing to pay to stay open. If it closes he loses 50; if it remains open he covers costs. The fixed cost could be higher and the response would not change.
The producer surplus is positive, so the owner would be willing to pay for continued production. Producing with a positive surplus contributes to the recovery of fixed costs. Depending on the size of the surplus, the fixed costs could be partially or wholly covered or the company could even make a profit.

What happens if the question about willingness to pay to avoid the closure was made to the owner before spending on the fixed factor? Would the producer surplus be the highest potential payment for avoiding closure? The answer is yes, and this maximum is now zero. Before starting the production, all the factors are variable and with the numbers of our example the surplus is zero. In general, if there are no fixed costs, the profit and producer surplus are identical.

More often the analyst needs to know the changes in the producer’s surplus rather than the absolute value of the surplus; for example the change caused because the price or the production costs have changed.

Consider the case of a firm operating in competitive markets of factors and products that supplies a good \( x \), with a well behaved production function, \( x = x(L, K) \), with a variable factor of production \( L \) with price \( w \) and a fixed factor \( K \) whose price is assumed to be one. The company’s profit in the short term is equal to:

\[
\pi = px(L) - wL - K. \tag{11.19}
\]

As a decision rule to start production we can say that the firm will not invest unless the producer’s surplus \((PS)\) is equal to or greater than the fixed cost:

\[
PS = px(L) - wL \geq K. \tag{11.20}
\]

In the short term, deriving (11.19) with respect to \( L \), we obtain the first order condition of profit maximization:

\[
p \frac{dx(L)}{dL} - w = 0, \tag{11.21}
\]

which indicates that, to maximize profit, we should further increase production as long as the value of the marginal productivity of the variable factor is greater than the price of that factor.

With a fixed factor, under standard assumptions, the marginal productivity of \( L \) will eventually decrease, so that for the given prices \((p, w)\) equality (11.21) must be met. The profit function depends therefore on the price of the good \((p)\) and the price factor \((w)\).
The profit equation (11.19) can be expressed as a value function, which depends indirectly on $p$ and $w$ (we use the same idea as in the case of the indirect utility function):

$$V(p, w) = px(p, w) - wL(p, w) - K. \quad (11.22)$$

The derivative of the function (11.22) with respect to the price of the good (a parameter for the competitive firm) shows the variation that occurs in the firm’s profit at the maximum. Therefore it should not be interpreted as a first order condition, but the effect on profits of a change in the price at the optimum.

$$\frac{\partial V}{\partial p} = x(p, w) + p \frac{\partial x}{\partial L} \frac{\partial L}{\partial p} - w \frac{\partial L}{\partial p} = x(p, w). \quad (11.23)$$

The increase in the benefit of an infinitesimal change in price is equal to the quantity sold by the firm (the envelope theorem) since, as the firm is located at the optimal production level, we know that:

$$\left( \frac{\partial x}{\partial L} - \frac{\partial L}{\partial p} \right) = 0. \quad (11.24)$$

The result obtained in (11.23) is very useful for calculating the change in producer surplus when the price changes, because we simply need to integrate the function $x(p, w)$, which is the supply function of the company, using as integration limits the initial and the final prices:

$$\Delta PS = \int_{p_0}^{p_1} x(p, w) \, dp. \quad (11.25)$$

Figure 11.1 shows the change in producer surplus when the price increases from $p_0$ to $p_1$. This change, represented by the area $P\text{b}a\text{p}_0$, coincides with the change in the firm’s profit. Note that for a very small price increase the surplus reduces to $x_0$; that is, to the results stated in (11.23).

Deriving expression (11.22) with respect to the factor price (exogenous to the firm), we obtain the variation that occurs in the firm’s profit when the factor price changes; just as happens with the price of a good, the idea is not to obtain a condition of maximization but to calculate the change in profits at the optimum when a parameter changes.

$$\frac{\partial V}{\partial w} = p \frac{\partial x}{\partial L} \frac{\partial L}{\partial w} - L(p, w) - w \frac{\partial L}{\partial w} = -L(p, w). \quad (11.26)$$
The variation in profits resulting from a change in the factor price is equal to the amount of a variable factor (with a negative sign) used by the company (the envelope theorem) because, given that the firm is maximizing profit, we know that:

\[
\left( \frac{\partial x}{\partial p} - w \right) \frac{\partial L}{\partial w} = 0. \tag{11.27}
\]

Integrating the demand function of the factor \( L(p, w) \), with a negative sign, between the initial and the final price factor, we obtain the following expression that gives the change in the producer surplus:

\[
\Delta PS = - \int_{w_0}^{w_1} L(p, w) \, dw. \tag{11.28}
\]

The representation of this change is reflected by the area \( w_0abw_1 \) in Figure 11.2.

Figure 11.1 shows the variation in producer’s surplus in the market for goods when the price changes, while Figure 11.2 represents the change in producer surplus on the market factor when the price of that factor changes. Alternatively the surplus arising from the change in the price of the good can be measured in the market for the factor, and the surplus arising from a change in the factor price can be measured in the market for goods. Figures 11.3 and 11.4 capture the areas of Figures 11.1 and 11.2 respectively.

An increase in the price from \( p_0 \) to \( p_1 \) in the product market (Figure 11.1) leads to an upward shift of the labour demand curve in Figure 11.3, which represents the value of the marginal productivity of labour. As the price of the factor (\( w \)) remains constant in \( w_0 \), the condition (11.21) is not satisfied, so more labour will be hired until the condition is satisfied. The change in
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producer surplus in the factor market equals the area $bdfa$, between the two curves of factor demand limited by the factor price that remains constant at the new equilibrium.

Similarly, a price reduction in the factor market (Figure 11.2) results in a downward shift of the supply curve in the product market (Figure 11.4), which represents the reduction in the marginal cost of production. As the price of the good is kept constant, reducing the marginal cost makes an increase in production profitable. The change in producer surplus in the market for goods equals the area $bdfa$, between the two supply curves of the good limited by the constant price at the new level of production.\(^5\)
Microeconomic foundations of cost–benefit analysis

Producer Surplus and the Surplus of the Owners of the Production Factors

The change in producer surplus as a result of the implementation of a project does not show who the beneficiaries of this surplus are. The producer surplus is a little confusing because eventually this change in the surplus may be a change in the income of the company owners, the owners of land and other fixed factors, the taxpayers or the workers.

Suppose factor $L$ is an imported raw material. In this case the reduction in the cost of production increases producer surplus and this is the change in social surplus (given that $p$ does not change, the consumer surplus remains constant). By contrast, if $L$ is domestic labour, the change in the surplus of workers has to be taken into account like the surplus of any other social agent. A reduction in $w$ as a result of the privatization of a public company represents a reduction in the workers’ surplus, which for constant $x$ offsets the increase in producers’ surplus, leaving social welfare unaffected.

Workers offer their labour if they are paid wages at least equal to their opportunity costs. Quite often, in the economic evaluation of projects, worker surplus obtained by subtracting the opportunity cost of working (for example the value of leisure) from the wages is not included, principally because many projects do not produce significant changes in the labour market, changing the equilibrium wage.

Figure 11.5 represents a function of labour supply $S_L(w)$. The minimum reservation wage is $w_r$, below which no one is willing to work in the market represented in the figure. This might be because there is an unemployment benefit or because the informal economy offers jobs paid around $w_r$.

Suppose that the labour market represented in Figure 11.5 is in
equilibrium at point $d$, so that at wage $w_0$, $L_0$ workers are employed. If the wage rises to $w_1$, the change from $d$ to $b$ generates new jobs (from $L_0$ to $L_1$). In the event that $b$ is an equilibrium, new employees are willing to work for their opportunity cost (area $dbL_1L_0$). However they receive as wages the area $abL_1L_0$, resulting in a surplus for the new employees equal to the area $abd$. Existing workers also benefit from the wage increase and their surplus goes from $w_0dw$ to $w_1adw$; or, which is the same, the surplus of existing workers increases in the area $w_1adw_0$.

It should be emphasized that we could ignore the concept of producer surplus and simply add up the surpluses of the social agents: landowners, shareholders, consumers, workers and taxpayers. Although any individual belongs to more than one category, this functional classification is useful in cost–benefit analysis.

In practice it is interesting to distinguish between fixed factors imposed by the nature that persists in the long term, such as the available land in a particular urban area or the agricultural land of particular characteristics, and fixed factors by law, such as some licences that restrict the entry of new suppliers into the market.

Although it is straightforward to sum the surpluses of the agents, including the owners of fixed factors, the use of the producer’s surplus can be suitable when information is scarce and one has to work with assumptions about what happens in the market at a very aggregate level.

Finally, a note of caution to avoid double counting. Suppose that Figure 11.1 represents an increase in the surplus of farmers as a result of rising agricultural prices. In assessing the change that has occurred in the market, we can use the producer’s surplus represented in Figure 11.1...
or the income of the landowners if we expect land rents to be updated to absorb the benefits of farmers, but not both.

11.4 COMPENSATING VARIATION, EQUIVALENT VARIATION AND CONSUMER SURPLUS

A price reduction or an increase in product quality allows the individual to reach a higher level of utility. The individual has improved, although we do not know the magnitude of improvement since the value that is assigned to both levels of utility is completely arbitrary.

In cost–benefit analysis we aim to go further and want to know the change in the social welfare of such individual improvements and this requires aggregating the changes in individual utilities. If the magnitude of the differences in utility is arbitrary (it only matters that the utility in one state is higher than in the other), it makes even less sense to add the changes in utility for different individuals.

One way out of these problems is to rely on the monetary valuation that individuals give to the utility change, and then add these individual values (remember that the aggregation of individual valuations requires, for conversion to social welfare, to weight them according to the marginal social utility of income).

The three money measures of changes in individual utility most frequently used to evaluate economic changes are the compensating variation, the equivalent variation and consumer surplus.

The compensating variation ($CV$) can be defined as the income that can be taken from the individual once the change occurs, leaving him at the same level of utility as before the change. The equivalent variation ($EV$) can be defined as the income to be given to the individual to attain the same level of utility reached with the change, but without the change (that is the amount of income that is equivalent to the change). The advantage of the $CV$ and the $EV$ is that they are money measures of the change in the individual’s utility. The consumer surplus ($CS$) can be defined as the sum of the willingness to pay of each unit consumed (the area under the demand function between zero and the last unit consumed) minus the total expenditure on the good. The change in $CS$ is therefore the difference in surplus before and after the change.

To see the theoretical justification of these money measures of utility, we start by minimizing the expenditure of an individual subject to a constant level of utility $\bar{U}$ (for simplicity of notation we omit the subscript $i$):

$$\min_{x_1, \ldots, x_n} \sum_{j=1}^{n} p_j x_j - \mu(U(x_1, \ldots, x_n) - \bar{U}).$$  \hspace{1cm} (11.29)
The first order conditions are:

\[ p_j - \mu \frac{\partial U}{\partial x_j} = 0, \quad j = 1, \ldots, n, \quad (11.30) \]

\[ U(x_1, \ldots, x_n) = \bar{U}. \quad (11.31) \]

Obtaining from (11.30) and (11.31) the values \( x_j \) that minimize (11.29), we obtain an expenditure function at the optimal \( e(P, \bar{U}) \) in which \( P \) is the vector of prices \((p_1, \ldots, p_n)\):

\[ e = \sum_{j=1}^{n} p_j x_j(P, \bar{U}) - \mu \left[ (U(x(P, \bar{U}))) - \bar{U} \right]. \quad (11.32) \]

Deriving this value function with respect to \( p_j \):

\[ \frac{\partial e}{\partial p_j} = x_j(P, \bar{U}) + \sum_{h=1}^{n} p_h \frac{\partial x_h}{\partial p_j} - \mu \sum_{h=1}^{n} \frac{\partial U}{\partial x_h} \frac{\partial x_h}{\partial p_j} = x_j(P, \bar{U}). \quad (11.33) \]

Expression (11.33) shows that the variation in expenditure when the price of a good is changed infinitesimally is equal to the compensated demand \( x_j(P, \bar{U}) \) because, at the optimum, according to (11.30):

\[ \left( p_h - \mu \frac{\partial U}{\partial x_h} \right) \frac{\partial x_h}{\partial p_j} = 0; \quad h = 1, 2 \ldots, n. \quad (11.34) \]

The demand function \( x_j(P, \bar{U}) \) is called compensated or Hicksian demand and the utility is constant, while with the market demand \( x_j(P, \bar{M}) \) income is constant. The compensated demand function shows how the demanded quantity changes when the price of the good changes, while the income is adjusted so that the utility remains constant.

Using the indirect utility function \( V(P, M) = \bar{U} \), where \( \bar{U} \) is the level of utility that minimizes the expenditure in (11.32), and two levels of utility for \( \bar{U} \), the one before the change \( U^0 \) and the one obtained after the change \( U^1 \), we see that the individual improves when the sign of (11.35) is positive and is worse off when the sign is negative:

\[ V(P^1, M^0) - V(P^0, M^0). \quad (11.35) \]

Expression (11.35) tells us the sign of the change, but it does not give us any information on the magnitude of the change.\textsuperscript{1} In cost–benefit analysis, it is not enough to know that some individuals improve and others become
worse off. If the money measure for taxpayers is a reduction of 1000 with the project, it is not enough for the decision maker to know that workers, employers and consumers will be better off. He also needs to know the magnitude of the improvement to compare it with the costs incurred, and to know whether the benefits of the beneficiaries of the project are high enough to offset the costs.

We need money measures of changes in the utility. The compensating variation \((CV)\) is one of them (see Jara-Díaz and Farah, 1988). Suppose that income remains constant and prices fall. In expression (11.36) we see how the \(CV\) measures the improvement, expressed in monetary terms, experienced by the individual:

\[
U^0 = V(P^0, M^0) = V(P^1, M^0 - CV). \tag{11.36}
\]

With initial prices and income \((P^0, M^0)\), the utility is equal to \(U^0\) (we are at the point of departure, before the change). Expression (11.36) shows the income that should be taken \((CV)\) from the individual in order to preserve the utility at the initial level in the new situation. If the change was an increase in prices, the \(CV\) would be negative (we would increase the income of the individual). Inverting (11.36):

\[
M^0 = e(P^0, U^0) \quad \text{and} \quad M^0 - CV = e(P^1, U^0), \tag{11.37}
\]

so that the \(CV\) can be expressed as:

\[
CV = e(P^0, U^0) - e(P^1, U^0), \tag{11.38}
\]

which in the case of independent goods can be expressed as:

\[
CV = \sum_{j=1}^{n} \int_{P_0^j}^{P_1^j} x_j(P, U^0) dp_j. \tag{11.39}
\]

In equation (11.38) the \(CV\) appears as the difference in the minimum amount of income required to achieve the level of utility \(U^0\) given the initial and final prices. If the difference is positive, the individual has improved (as you can take income to leave him at the same level of utility). If the difference is negative, the individual is worse off because we have to give him income to compensate for the change.

The \(CV\) in (11.39) is valid for the case of independent goods and uses the result in (11.33), where the derivative of the expenditure at the optimum with respect to price is the compensated demand function \(x_j(P, U^0)\). If we integrate the compensated demand function, using the initial and final
prices as integration limits, we obtain an area equal to the $CV$, since the compensated demand does not incorporate the income effect.

We can proceed in the same way for the equivalent variation:

$$U^1 = V(P^1, M^0) = V(P^0, M^0 + EV). \quad (11.40)$$

With the income constant and the final prices $(P^1, M^0)$ the utility is equal to $U^1$ (the level of utility after the change). Suppose that at this utility level $U^1$ the individual has improved because the prices $P^1$ are lower than $P^0$. Expression (11.40) shows that the $EV$ consists of giving income to the individual in order, with the initial prices $(P^0)$ and without the change, to make him as well off as in the level of utility $U^1$, equivalent to the change. If the change was an increase in prices, the $EV$ would be negative (income will be taken from the individual). Inverting (11.40):

$$M^0 = e(P^1, U^1) \quad \text{and} \quad M^0 + EV = e(P^0, U^1), \quad (11.41)$$

and the $EV$ can be expressed as:

$$EV = e(P^0, U^1) - e(P^1, U^1), \quad (11.42)$$

which in the case of independent goods can be expressed as:

$$EV = \sum_{j=1}^{n} \int_{p_j^0}^{p_j^1} x_j(P, U^1) \, dp_j. \quad (11.43)$$

In expression (11.42), the $EV$ appears as the minimum income required for attaining the level of utility $U^1$ with the initial and final prices. If the difference is positive, the individual has improved (as an increase in income is equal to a change that leads to a higher level of utility). If the difference is negative, the individual is worse off since the change is equivalent to a loss of income.

In the same way as with the $CV$, expression (11.43) is only valid for independent goods and uses the result in (11.33) where the derivative of the expenditure in the optimum with respect to price is the compensated demand function, though now at a different level of utility $x_j(P, U^1)$. By integrating the compensated demand function for a level of utility $U^1$ and using the initial and final price of the good $j$ as limits of integration, an area equal to the $EV$ is obtained.

The $CV$ and the $EV$ do not necessarily coincide. They both measure changes in the value of the individual’s utility in terms of money, with respect to the initial level in the case of $CV$ and the final level in the case of $EV$. 
Figure 11.6  Why do CV and EV not coincide?

Figure 11.6 represents, in the upper part, the indifference curves and the budget constraint of a consumer who chooses between good \( x_1 \) (in the vertical axis) and other goods, or income (in the horizontal axis). Initially, the individual is located at point \( b \), where the indifference curve \( U^0 \) is tangent to the budget constraint \( zM_2 \). Their income level is equal to \( M_2 \), which is the maximum amount of other goods that the individual could consume if he does not consume \( x_1 \). Suppose that we reduce the price of \( x_1 \), shifting the budget constraint to \( hM_2 \), and now the individual chooses point \( l \). The CV consists of taking away income from the individual such that he reaches the same level of utility enjoyed in \( b \) but once the price of \( x_1 \) has changed. Point \( d \) is the new location, and is achieved by reducing income by \( M_2 - M_1 \). Applying the definition of EV, we should give income to the individual at
the initial price in order to obtain the same level of utility as at point \( f \) (point \( l \)), which is achieved with an income increase of \( M_1 - M_2 \).

Figure 11.6 allows us to link the movements of income of \( CV \) and \( EV \) in the upper part of the figure with changes in the utility and its representation as areas in the bottom part. Let us look at this in more detail.

As can be observed, changes in income are not equal on the horizontal axis \((M_2 - M_1)\) \( (M_3 - M_2) \); however changes in utility are identical: from \( U^1 \) to \( U^0 \) in the \( CV \) and from \( U^0 \) to \( U^1 \) in the \( EV \). If the changes in utility are identical, why are changes in income not equal? The bottom part of the figure represents in the vertical axis the marginal utility of income, and in the horizontal axis the income. The area between the different levels of income and the curve, which represents the marginal utility of income, is the change in utility due to changes in income.

Under the assumption of decreasing marginal utility of income, which implies that when the income is lower its marginal utility is higher and vice versa, areas \( A \) and \( B \) must be equal, because these areas represent the change in utility from \( U^1 \) to \( U^0 \) and from \( U^0 \) to \( U^1 \) (integral defined between the income levels of the corresponding curve of marginal utility of income). The absolute magnitude of the income is lower when the individual moves to a lower level of income (from \( M_2 \) to \( M_1 \)) than when moving to a higher one (from \( M_2 \) to \( M_3 \)).

Figure 11.7 shows a compensated demand function and a reduction in the price of the good \( x_j \) from \( p_0 \) to \( p_1 \). The area between the two prices and the compensated demand can be the compensating variation or the equivalent variation depending on which level of utility is used as a reference (initial or final). In the special case of zero income effect the area is common to the \( CV \), the \( EV \) and the change in \( CS \), which we address below.

From the market demand function \( x(P, \bar{M}) \) we can calculate the change in consumer surplus as a result of a change in the price of one or several
goods. Let $p_0$ be the initial price of good $j$ and $p_1$ the final price; since consumer surplus is the difference between what individuals are willing to pay and what they actually pay, the change in consumer surplus ($\Delta CS$) as a result of the change in prices is equal to:

$$\Delta CS = \sum_{j=1}^{n} \int_{p_j^0}^{p_j^1} x_j(P, \bar{M}) dp_j. \quad (11.44)$$

The change in consumer surplus presents several problems; for example we do not always obtain a single number because if several prices change, or prices and income change, the result depends on the path of integration. Another drawback of expression (11.44) is that, although consumer surplus has a unique value, regardless of the order in which the changes in prices are integrated, it does not measure changes in the utility unless the marginal utility of income is constant. Consider this last point in more detail. From (11.13) we know that

$$x_j = \frac{\partial V}{\partial p_j}, \quad (11.45)$$

and substituting in (11.44):

$$\Delta CS = -\sum_{j=1}^{n} \int_{p_j^0}^{p_j^1} \frac{\partial V}{\mu \partial p_j} dp_j. \quad (11.46)$$

If we want to relate the change in $CS$ to changes in the utility, it is required that the marginal utility of income is constant. If $\mu$ is not constant, it can even happen that the change in $CS$ has a different sign to the change in utility if the price changes have different signs (see Just et al., 1982). The representation of expression (11.44) for the change in one price is the area between the initial price and the final price and the market demand function (see Figure 11.8).

**Market Demand and Compensated Demand**

In most economic evaluations of projects, market demand functions are required. Market demand allows us to calculate changes in consumer surplus and even the total surplus if there is available information to approximate the maximum reservation price. The use of the market demand function to estimate the change in the welfare of individuals has the disadvantage of including the income effect.
Market demand is the horizontal sum of individual demands. Figure 11.8 shows a consumer maximizing his utility, with the good $x$ on the horizontal axis and the other goods (or income) on the vertical axis. Located initially at point $a$ he consumes the quantity $x_a$ of good $x$ and $M_1$ of the other goods, reaching a level of utility $U_0$, represented by the indifference curve. The bottom part of the figure represents the inverse market demand function of good $x$, $x(p, M_0)$. Point $a$ in the bottom part of the figure shows that at price $p_0$ the consumer demands the quantity $x_a$.

Consider that, as a result of launching a public project, the price of good $x$ changes from $p_0$ to $p_1$, which is represented in Figure 11.3 by reducing the slope of the budget constraint (which goes from (1) to (2)) while maintaining the same intersect on the vertical axis (the price of other goods does not change).

How has the individual improved with the change? One way of answering this question is to ask the consumer how much money he would be
willing to pay in order to leave him at the level of utility at which he was located before the change in price (\(U^0\) indifference curve). Recall that this concept is the compensating variation.

The problem with the above question is that indifference curves are not observable. It is difficult and expensive to obtain the information through consumer surveys, especially if there is an observable ordinary demand function in the market from which we can estimate the improvement. Can we get the measurement of the improvement with the consumer demand observed in the market? Consider two possible equilibria (\(d\) and \(b'\)) once the change in price happens and the consumer adjusts his basket of consumption. The reason why the individual chooses point \(d\) or \(b'\) depends entirely on his preferences, since both points are on the budget constraint line.

Suppose that \(d\) is the point through which the new indifference curve is tangent to (2), increasing consumption of good \(x\) to \(x_d\) and reducing the consumption of other goods. At the bottom of the figure, the market demand function shows the shift from \(a\) to \(d\) as a result of reducing the price to \(p_1\). The change in consumer surplus equals the area \(p_0adp_1\). Compare this surplus with the compensating variation, taking away income from the individual in the upper part of the figure to leave him indifferent to the situation prior to the completion of the project (point \(b\) and quantity \(x_b\)). We leave the individual with only the substitution effect (budget constraint line (3) parallel to (2)) by removing the income effect. The demand function that goes from \(a\) to \(b\) is the compensated demand (without income effect) and the area \(p_0abp_1\) measures the compensating variation. What does the area \(adb\) represent?

To answer this question, imagine that the new indifference curve tangent to (2) passed through \(b'\) instead of \(d\). The quantity chosen is now \(x_{b'}\). In order to obtain the compensating variation we would lead the individual to point \(b\) (the same as above), but now we see that the functions of market demand and compensated demand coincide. The explanation is in the income effect. When the individual is at \(b\) after taking away income from his budget (regardless of whether he comes from \(d\) or \(b'\)), the improvement experienced by the individual as a result of the price reduction is measured by the area \(p_0abp_1\).

This is the improvement that we want to measure and not what the consumer is going to spend this improvement on. When the individual chooses \(b'\) the income effect derived from the improvement is entirely dedicated to good \(M\) (the income effect in good \(x\) is zero); but if the individual spends part of the improvement represented in the area \(p_0abp_1\) on consuming more \(x\) (at point \(d\) with quantity \(x_d\)), the area of the demand function \(p_0adp_1\) would overestimate the change in the welfare of the individual by the amount \(adb\), which is simply part of the area \(p_0abp_1\) now spent on good \(x\) rather than on other goods.
The Magnitude of the Error

In the practice of cost–benefit analysis it is unusual to work with the compensated demand function. It has been shown that the bias resulting from the use of the ordinary demand curve can be insignificant when the expenditure on the good or goods affected by the project does not represent a high percentage of the budget of the individual, in which case the consumer surplus estimated with the ordinary demand is a good approximation in most cases (see Willig, 1976). On the other hand it should not be forgotten that in the evaluation of the project we only know the initial price and the initial quantity; therefore the price reduction and the demand response are only estimates. Errors associated with poor data quality are probably more important than those derived from calculating the change in consumer utility in money terms with the market demand instead of the compensated demand.

Let us very briefly analyse why we can use consumer surplus as a monetary measure of changes in the welfare of individuals. Returning to Figure 11.8, where the ordinary demand \( x(p, M^0) \) is represented, we want to evaluate the improvement experienced by the consumer after the price drops from \( p_0 \) to \( p_1 \). The change in consumer surplus (\( \Delta CS \)) is equal to the area \( p_0adp_1 \) and the compensating variation is equal to the area \( p_0abp_1 \). The difference between the two areas is \( adb \). This is the overestimation of the benefits if we use \( \Delta CS \) instead of the \( CV \). What is the magnitude of this error?

The area \( adb \) is triangular and for small price changes it is approximately equal to the area \( 1/2 \Delta p \Delta x \), where \( \Delta p \) is the change in the price and \( \Delta x \) is the income effect (shift from \( x_b \) to \( x_d \)).

The elasticity of demand with respect to income is:

\[
\eta = \frac{dx}{dM} \frac{M}{x}.
\]  
(11.47)

For small changes:

\[
\Delta x = \eta x \frac{\Delta M}{M},
\]  
(11.48)

and also \( \Delta M \approx \Delta CS \). Substituting into (11.48):

\[
\Delta x = \eta x \frac{\Delta CS}{M},
\]  
(11.49)

so that the area \( adb \) is equal to:
\[
\frac{1}{2} \Delta p \Delta x = \frac{1}{2} \eta \Delta CS x \frac{\Delta p}{M}. \quad (11.50)
\]

The CV is equal to the change in consumer surplus minus the overestimation represented by the area \( a \) \( b \). We know that for a small change in the price \( \Delta CS \approx x \Delta p \). Solving for \( \Delta p \) and substituting into (11.50):

\[
CV = \Delta CS - \frac{\eta}{2M} (\Delta CS)^2. \quad (11.51)
\]

Operating in (11.51):

\[
\frac{CV - \Delta CS}{\Delta CS} = -\frac{1}{2} \frac{\Delta CS}{\eta} \frac{\Delta CS}{M}. \quad (11.52)
\]

According to (11.52), the relative error of using the CS instead of the CV is low if \( \eta \) or \( \Delta CS/M \) are low enough. For example, with a high income-demand elasticity (\( \eta = 2 \)) and \( \Delta CS/M \leq 0.05 \), or for \( \eta = 1 \) and \( \Delta CS/M \leq 0.1 \), the error is less than or equal to 5 per cent.

In the case of non-marketed goods, the compensating and equivalent variations are quite useful and their estimation is carried out with consumer surveys through which the analyst tries to measure the willingness to pay or willingness to accept of an impact that changes the utility of the individual.

Theoretically these measurements do not bear the drawbacks of consumer surplus; however the responses that individuals give to the interviewer are not necessarily the true \( EV \) or \( CV \) if the individual has no preferences (is able to realize the trade-off) on this kind of goods, or the individual suspects that his response may influence the outcome of the evaluation. Furthermore we are implicitly assuming that the individual fully understands the questions in the survey, and that what is asked exactly reflects what we want to know, among other possible problems (see Chapters 5 and 6).

11.5 UNCERTAINTY

Assuming that consumers and firms maximize their utility, their choices are taken in relation to the results that are associated with these decisions. The problem is that there are often several possible outcomes associated with the same decision. The purchase of shares in a company can produce profits or losses for shareholders depending on the 'states of nature'; that
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is, the circumstances in the world that affect the profitability of the shares and in which the shareholder is unable to intervene.

The uncertainty is associated with the existence of different possible states of nature. If the individual buys a financial asset whose profitability depends on the state of nature, the risk is present in his decisions. The challenge now is to make decisions that involve risk.

Most individuals do not like the variability in the results. They are risk averse and they typically buy insurance to ensure a stable pattern of consumption (or profits) not subject to the uncertain states of nature.

An action subject to risk is the investment in infrastructure. Long life, specific assets, sunk costs and uncertain demand make the net present value of these investments very difficult to predict. If the demand rises or falls over the life of the investment project, profitability can change dramatically.

There are a number of useful concepts for the analysis and understanding of the economic consequences of uncertainty: decreasing marginal utility, expected value, utility of the expected value, expected utility and certainty equivalent. Figure 11.9 can help with the definitions and the subsequent explanation.

The total utility \( U \) represented in Figure 11.9 is increasing and positive. When the individual receives income \( M \) on the horizontal axis his utility is positive; when he receives more income his utility increases. The concavity of the total utility curve adds a third assumption, that the utility increases with income but less than proportionately; that is, the marginal utility of income is positive but decreasing.

![Figure 11.9  Uncertainty](image-url)
Positive and increasing income and total utility mean that the higher the income the happier the individual. Positive and diminishing marginal utility means that when you receive additional income, your happiness increases more when you are ‘poor’ than when you are ‘rich’. Suppose the individual whose utility curve is represented in Figure 11.9 is offered, for no payment, the following game: a coin is thrown; if it is heads he wins a million dollars, if it is tails he wins nothing. Before throwing the coin he is offered an amount of money for not playing. The individual may abandon the game and accept the money or reject the money and play.

The expected value of the game is equal to half a million dollars \((\frac{1}{2} \times 0 + \frac{1}{2} \times 1)\) but if the individual plays this result will never occur. The result will be either heads and he wins a million, or tails and he wins nothing. The expected value (0.5) is the approximate result that would occur if the game is repeated a sufficiently large number of times; but the individual plays only once. What is the minimum amount we should offer him for not playing?

The minimum we would have to offer (his reservation price for not playing) should correspond to a similar level of utility to the one attained if he accepts the game. It is the amount of income that leaves the individual indifferent between playing and not playing. This level of income varies among individuals and is related, in addition to the initial level of income, to the degree of risk aversion.

Individuals with similar incomes may have different reservation prices. In the case of the individual in Figure 11.9 the reservation price is $200,000. If he is offered a lower amount, he will play. The value 0.2 on the horizontal axis corresponds to a level of utility that is identical to the level he would achieve if he accepts the game.

It should be emphasized that the level of utility of accepting the game is an expected level of utility \((\frac{1}{2}U(0) + \frac{1}{2}U(1))\). This expected level of utility corresponds to the option ‘accept the game’ and not the utility obtained after playing it, which is either \(U(0)\) or \(U(1)\). The individual is indifferent between playing the game with the same probability of obtaining the level of utility \(U(0)\) or \(U(1)\) and not playing if the offer is at least $200,000. This amount of money, which gives the same level of utility as the one obtained by playing the game, is denominated the ‘certainty equivalent’.

It can be seen that the risk aversion of the individual makes the utility of the expected value \(U(0.5)\) higher than the expected utility \((\frac{1}{2}U(0) + \frac{1}{2}U(1))\). This individual would be willing to pay a maximum of $800,000 for insurance before playing. If he ensures the guarantee of 1 million dollars whatever the result of the coin tossing, he will always gain the level of utility 0.2. If it is heads, he wins 1 million, less the premium of 0.8, he has 0.2. If it is
tails he has zero but the insurance will compensate him with 1, and this 1 minus the premium of 0.8 is again 0.2.

As the expected value is what would be obtained if the game is repeated many times, an insurance company (which might also be thought of as an agreement among the large group of individuals identical to the one represented in Figure 11.9) could offer an insurance policy at a premium for which it would have to charge each individual at least 0.5 (which is called a fair premium\(^{10}\)), and the individuals would pay a maximum of 0.8.

NOTES


2. Note that expression (11.2) does not specify the procedure for aggregating the benefits of the \( m \) individuals. It is compatible with any external criterion imposed by the analyst.

3. The derivative of the value of an objective function at the optimum with respect to an exogenous parameter is equal to the derivative of the objective function with respect to the parameter. Only the direct effect must be considered.

4. Utility, and profits, can also be affected through a change in the level of an environmental good (e.g. air quality). This is covered in Chapters 5 and 6.

5. The previous analysis is generalized for multiple goods and factors (see Just et al., 1982).

6. To define the \( CV \) as the income ‘taken’ or ‘given’ is arbitrary. Here it is defined as the amount of income to be taken from the individual after the change to bring him back to the initial level of utility, so an increase in utility will be associated with a \( CV \) with a positive sign, whereas a loss of utility will have a \( CV \) with a negative sign (‘taken’ with a negative sign is the same as ‘given’).

7. Recall that any monotonic increasing transformation of the original utility function is also valid. If a utility function, which represents the preferences of an individual before the project is implemented, has a value of \( U^0 = 1 \) and after the project \( U^1 = 3 \), we know that the individual has improved. If transforming the original function, the new values were \( U^0 = 1 \) and \( U^1 = 9 \), the economic interpretation has not changed. The ranking of the basket of goods of the individual is identical. In \( U^1 \) the individual is better off than in \( U^0 \), but we do not know the magnitude of the change because the scale is arbitrary.


9. Although the individual enters the game with a positive level of income and a positive level of total utility, we assume for simplicity, as Figure 11.9 shows, that \( U(0) \) is the point corresponding to his original position before playing the game.

10. Assuming that there are no operating costs of the insurance company or transaction costs between individuals to reach an agreement and compliance are zero.
References


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